New perturbative method of solving the gravitational N-body problem in general relativity

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Presenting a new perturbative method to deal with the gravitational N-body problem in general relativity. A novel approach to construct global and local reference frames leads to explicit transformation rules for metric tensor and related relativistic gravitational potentials as well as efficient formulation of the equations of motion.

Recent experiments have successfully tested Einstein's general theory of relativity in a variety of ways and to remarkable precision. These experiments included spacecraft Doppler tracking, planetary radar ranging, lunar and satellite laser ranging, as well as a number of dedicated gravitational experiments in space and many ground based efforts. How can computational models keep up with the ever improving accuracy of these missions?

Finding a solution to the Einstein's gravitational field equations in the case of an unperturbed one-body problem is quite a simple task. A generalization of the resulting post-Newtonian solution to a system of N extended arbitrary bodies is not so straightforward. The coupling of the intrinsic multipole moments or the angular momentum of an extended or spinning body to the gravitational field affects the body's equations of motion. The transformation of these quantities from one coordinate frame to another must take into account the nonlinearity of the the gravitational interaction.

Although the theory of relativity is independent of any coordinate representation, picking the right coordinate chart can greatly expedite calculations. In our approach, we break down the N-body problem using a global inertial frame (e.g., a barycentric reference frame for the system being studied) and a local (noninertial) reference frame for each of the N bodies, to study the structure and gravitational properties of that body. Finding the right transformation rules between these reference frames is an essential prerequisite to finding the relativistic equations of motion for the system.

In this approach, the solution to the gravitational field equations in any reference frame is presented as a sum of three terms: (i) The inertial flat spacetime in that frame, (ii) unperturbed solutions for each body in the system transformed to the coordinates of this frame and (iii) the gravitational interaction term. The harmonic gauge conditions and dynamical conditions are used to construct the local reference frame associated with each body. This method has the unique advantage that it allows us to develop explicit forms for both direct and inverse coordinate transformations simultaneously and in a similar manner.

The results, accurate to the second post-Newtonian order, address the needs of practical astronomy and high-precision experiments. These results will find immediate use in many areas of modern geodesy, astronomy and astrophysics. The approach can also be extended in an iterative, if greater accuracy is desired, by including higher-order terms. In addition, the same approach can be used to develop a theory of relativistic rotating reference frames in general relativity.

Our current efforts are directed towards the practical application of the results, similar to our work on the Gravity Recovery and Interior Laboratory (GRAIL) and Gravity Recovery and Climate Experiment Follow-on (GRACE-FO) missions. We also plan to address the case of relativistic rotation and a complete formal treatment of N extended bodies. The first of these is especially important as modern observational accuracy of the geodynamical observations makes it necessary to have a rigorous relativistic model of the Earth's rotation.

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