New study predicts variation in illness severity in a population

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Many of us are familiar with bell-shaped curves that describe the distributions of school grades and total annual rainfall, among many other quantities. This ubiquitous distribution results when many points for individual non-correlated quantities are added to produce an outcome.

Interestingly, a very differently-shaped, highly skewed pattern, often called a power law distribution, is also ubiquitous. This skewed distribution is often considered a signature of complex systems, but its origin has never been adequately explained.

A new study shows that when individual random quantities, such as reactions in the body, are instead correlated, and multiplied, the process gives rise to the ubiquitous, highly skewed pattern, and that this pattern is more accurately called a Weibull distribution. It goes on to show that the Weibull distribution can be used to describe and predict the pattern of illness severity across a population exposed to different chemicals.

The correlated multiplicative model can potentially explain many types of results in a wide range of areas, and thus lead to better predictions of many types of risk, including human health risk.

"The insight is new and cuts across scientific fields," said James D. Englehardt, professor of Environmental Engineering at the University of Miami (UM) College of Engineering and author of the study. "The correlated multiplicative, or first-order kinetic, model can explain many different processes, ranging from mortality rates to distributions of incomes, material released into the environment during oil spills, explosions and natural disasters to the magnitudes of solar flares, among many other quantities."

The Weibull distribution is similar to a power-law, which has been observed to describe many aspects of complex systems. But according to the study, the Weibull distribution is superior in explaining these systems.

"In the case of a power law probability distribution, the probability of an outcome size occurring is proportional to a power of the size," Englehardt said. "However, as a probability distribution, it must necessarily break down at one or both extremes, otherwise over the range zero to infinity there would be a total probability of all possible outcomes greater than one," he said. "A Weibull distribution does not have this problem," he said. "It can fit the full range of observed data, and the paper shows it to have better 'goodness-of-fit' to illness severity data, and to be predicted theoretically in many complex systems."

Previous studies have not generally used the Weibull form in place of the power law to describe complex systems, in part because the sizes of the individual multiplicative causes of complex systems outcomes are not often known, and in part because some of these systems are not often analyzed numerically. For instance, illness severities are described by clinical findings, which are generally sets of observed symptoms leading to a yes/no diagnosis.

"Another more technical reason is that the 'first-order kinetic model,' used in economics, chemistry, biology, and pathology to describe how many quantities grow or decay over time, has not been recognized to describe the sizes of outcomes of many complex systems, and has not been generally recognized as equivalent to a multiplicative model, and the incremental multipliers of that model have not generally been recognized as correlated and exponentially-distributed, though it is known that products of perfectly-correlated exponential random variables are Weibull," Englehardt said.

The study, titled "Distributions of Autocorrelated First-Order Kinetic Outcomes: Illness Severity," is published in the online journal *PLOS ONE*. Currently, Englehardt is working on a general dose-response function for chemicals and chemical mixtures, based on the Weibull illness severity distribution that may greatly improve our ability to predict the risk of illness resulting from exposure to extremely low doses of chemicals that we experience in our daily lives.

More information: *PLOS ONE*, journals.plos.org/plosone/arti … journal.pone.0129042

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