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Mathematicians prove the Umbral Moonshine Conjecture

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In theoretical math, the term "moonshine" refers to an idea so seemingly impossible that it seems like lunacy.

Monstrous moonshine, a quirky pattern of the monster group in theoretical math, has a shadow - umbral moonshine. Mathematicians have now proved this insight, known as the Umbral Moonshine Conjecture, offering a formula with potential applications for everything from number theory to geometry to quantum physics.

"We've transformed the statement of the conjecture into something you could test, a finite calculation, and the conjecture proved to be true," says Ken Ono, a mathematician at Emory University. "Umbral moonshine has created a lot of excitement in the world of math and physics."

Co-authors of the proof include mathematicians John Duncan from Case Western University and Michael Griffin, an Emory graduate student.

"Sometimes a result is so stunningly beautiful that your mind does get blown a little," says Duncan, who co-wrote the statement for the Umbral Moonshine Conjecture with Miranda Cheng, a mathematician and physicist at the University of Amsterdam, and Jeff Harvey, a physicist at the University of Chicago.

Ono will present their work on January 11, 2015 at the Joint Mathematics Meetings in San Antonio, the largest mathematics meeting in the world. Ono is delivering one of the highlighted invited addresses.

Ono gave a colloquium on the topic at the University of Michigan, Ann Arbor, in November, and has also been invited to speak on the umbral moonshine proof at upcoming conferences around the world, including Brazil, Canada, England, India, and Germany.

It sounds like science fiction, but the monster group (also known as the friendly giant) is a real and influential concept in theoretical math.

Elementary algebra is built out of groups, or sets of objects required to satisfy certain relationships. One of the biggest achievements in math during the 20th century was classifying all of the finite simple groups. They are now collected in the ATLAS of Finite Groups, published in 1985.

"This ATLAS is to mathematicians what the periodic table is to chemists," Ono says. "It's our fundamental guide."

And yet, the last and largest finite simple group, the monster group, was not constructed until the late 1970s. "It is absolutely huge, so classifying it was a heroic effort for mathematicians," Ono says.

In fact, the number of elements in the monster group is larger than the number of atoms in 1,000 Earths. Something that massive defies description.

"Think of a 24-dimensional doughnut," Duncan says. "And then imagine physical particles zooming through this space, and one particle sometimes hitting another. What happens when they collide depends on a lot of different factors, like the angles at which they meet. There is a particular way of making this 24-dimensional system precise such that the monster is its symmetry. The monster is incredibly symmetric."

"The monster group is not just a freak," Ono adds. "It's actually important to many areas of math."

It's too immense, however, to use directly as a tool for calculations. That's where representation theory comes in.

Shortly after evidence for the monster was discovered, mathematicians John McKay and John Thompson noticed some odd numerical accidents. They found that a series of numbers that can be extracted from a modular function and a series extracted from the monster group seemed to be related. (One example is the strange and simple arithmetic equation 196884 = 196883 + 1.)

John Conway and Simon Norton continued to investigate and found that this peculiar pattern was not just a coincidence. "Evidence kept accumulating that there was a special modular function for every element in the monster group," Ono says. "In other words, the main characteristics of the monster group could be read off from modular functions. That opened the door to representation theory to capture and manipulate the monster."

The idea that modular functions could tame something as unruly as the monster sounded impossible - like lunacy. It was soon dubbed the Monstrous Moonshine Conjecture.

(The moonshine reference has the same meaning famously used by Ernest Rutherford, known as the father of nuclear physics. In a 1933 speech, Rutherford said that anyone who considered deriving energy from splitting atoms "was talking moonshine.")

In 1998, Richard Borcherds won math's highest honor, the Fields Medal, for proving the Monstrous Moonshine Conjecture. His proof turned this representation theory for the monster group into something computable.

Fast-forward 16 years. Three Japanese physicists (Eguchi, Ooguri and Tachikawa) were investigating a particular kind of string theory using a particle physics model from the Mathieu Group M24, another important finite simple group.

"They conjectured a new way to extract numbers from the Mathieu Group," Duncan says, "and they noticed that the numbers they extracted were similar to those of the monster group, just not as large." Terry Gannon, a mathematical physicist, proved that their observations are true.

It was a new, unexpected analogue that hinted at a pattern similar to monstrous moonshine.

Duncan started investigating this idea with physicists Cheng and Harvey. "We realized that the Mathieu group pattern was part of a much bigger picture involving mock modular forms and more moonshine," Duncan says. "A beautiful mathematical structure was controlling it."

They dubbed this insight the Umbral Moonshine Conjecture. Since they published the final version of the more than 100-page conjecture online last June, it has been downloaded more than 2,500 times.

The conjecture caught the eye of Ono, an expert in mock modular forms, and he began pondering the problem along with Griffin and Duncan.

"Things came together quickly after the statement of the Umbral Moonshine Conjecture was published," Ono says. "We have been able to prove it and it is no longer a guess. We can now use the proof as a completely new and different tool to do calculations."

Just as modular forms are "shadowed" by mock modular forms, monstrous moonshine is shadowed by umbral moonshine. (Umbra is Latin for the innermost and darkest part of a shadow.)

"The job of a theoretical mathematician is to take impossible problems and make them tractable," Duncan says. "The shadow device is one valuable tool that lets us do that. It allows you to throw away information while still keeping enough to make some valuable observations."

He compares it to a paleontologist using fossilized bones to piece together a dinosaur.

The jury is out on what role, if any, umbral moonshine could play in helping to unravel mysteries of the universe. Aspects of it, however, hint that it could be related to problems ranging from geometry to black holes and quantum gravity theory.

"What I hope is that we will eventually see that everything is unified, that monstrous moonshine and umbral moonshine have a common origin," Duncan says. "And part of my optimistic vision is that umbral moonshine may be a piece in one of the most important puzzles of modern physics: The problem of unifying quantum mechanics with Einstein's general relativity."

Provided by Emory University

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