Insightful mathematics for an optimal run: Mathematical equations can help improve athletic performance

October 27 2014

Sure, we can become better runners by hydrating well, eating right, cross training, and practice. But getting to an optimal running strategy with equations? That's exactly what a pair of mathematicians from France propose in a paper published this month in the *SIAM Journal on Applied Mathematics*.

"By modeling running in the form of equations and then solving them, we can predict the optimal strategy to run a given distance in the shortest amount of time," says Amandine Aftalion, who co-authored the paper with Frederic Bonnans.

The model uses a system of ordinary differential equations. Aftalion explains: "Our model relies on two basic principles: energy is preserved, and acceleration (or variations of velocity) is equal to the sum of all forces. This leads to a system of differential equations coupling the unknown variables of the runner (velocity, propulsive force and anaerobic energy), and dependent on physiological parameters such as maximal oxygen uptake and total available anaerobic energy."

These equations are utilized to deduce the optimal running strategy using numerical simulations and rigorous analysis.

"The difficulty in numerical simulation is that the equations are coupled, hence, none of the variables can be solved independently of the others.

Using an optimal control solver developed by Inria (The French Institute for Research in Computer Science and Automation), we are able to get a full numerical solution, something that hasn't been done previously," says Aftalion.

A well-know study conducted by Joseph Keller in 1974 concluded that running a race requires an athlete to keep an almost constant speed for optimal effectiveness. Keller assumed that a runner keeps his or her maximal oxygen uptake at a constant value, whereas this value is seen to gradually increase to its maximal value during a race and then drop off at the end. In this paper, Aftalion and Bonnans argue that physiological measurements demonstrate that runners do not keep a constant speed, usually tending to vary their speed by an order of 10%. The authors show that varying one's velocity rather than running at a constant velocity allows one to run longer. This is done using optimal control theory.

Performance data from races helps in assessing this new model. "We are now able to identify the physiological parameters of a person through data of a good race on a given distance using several time measurements at regular distances in the race," says Aftalion. "From this we can predict how to run an ideal race, both for a champion, helping him improve his performance and win a medal, as well as for a regular runner who lacks professional coaching and seeks help. Our predictions corroborate actual strategies used by professional athletes."

"In the future, we plan to adapt our model to other sports such as biking, triathlons, or other endurance sports, maybe cross country skiing," Aftalion says.

More information: Optimization of running strategies based on anaerobic energy and variations of velocity, *SIAM Journal on Applied Mathematics*, 74(5), 1615–1636 (Online publish date: October 9, 2014). The paper is available for free download at the link above until January

27, 2015.

Provided by Society for Industrial and Applied Mathematics

Citation: Insightful mathematics for an optimal run: Mathematical equations can help improve athletic performance (2014, October 27) retrieved 13 May 2024 from https://phys.org/news/2014-10-insightful-mathematics-optimal-mathematical-equations.html

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