

Mathematicians trace source of Rogers-Ramanujan identities, find algebraic gold

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Mathematicians have found a framework for the celebrated Rogers-Ramanujan identities and their arithmetic properties, solving another long-standing mystery stemming from the work of Indian math genius Srinivasa Ramanujan.

The findings, by mathematicians at Emory University and the University of Queensland, yield a treasure trove of algebraic numbers and formulas to access them.

"Algebraic numbers are among the first numbers you encounter in mathematics," says Ken Ono, a number theorist at Emory "And yet, it's surprisingly difficult to find functions that return them as values in a uniform and systematic way."

Ono is the co-author of the new findings, along with S. Ole Warnaar of the University of Queensland and Michael Griffin, an Emory graduate student.

The most famous algebraic number of all is the [golden ratio](#), also known by the Greek letter phi. Many great works of architecture and art, such as the Parthenon, are said to embody the pleasing proportions of the golden ratio, which is also seen in beautiful forms in nature.

Mathematicians, artists and scientists, from ancient times to today have pondered the qualities of phi, which is approximately equal to 1.618, although its digits just keep on going, with no apparent pattern.

"People studied the golden ratio before there was a real theory of algebra," Ono says. "It was a kind of prototype for algebraic numbers."

Although no other algebraic units are as famous as the golden ratio, they are of central importance to algebra. "A fundamental problem in mathematics is to find functions whose values are always algebraic numbers," Ono says. "The famous Swiss mathematician Leonhard Euler made some progress on this problem in the 18th century. His theory of continued fractions, where one successively divides numbers in a systematic way, produces some very special algebraic numbers like the golden ratio. But his theory cannot produce algebraic numbers which go beyond the stuff of the quadratic formula that one encounters in high school algebra."

Ramanujan, however, could produce such numbers, and he made it look easy.

"Ramanujan has a very special, almost mythic, status in mathematics," says Edward Frenkel, a mathematician at the University of California, Berkeley. "He had a sort of Midas touch that seemed to magically turn everything into gold."

And the Rogers-Ramanujan [identities](#) are considered among Ramanujan's greatest legacies, adds Frenkel, a leading expert on the identities.

"They are two of the most remarkable and important results in the theory of q-series, or special functions," says Warnaar, who began studying the Rogers-Ramanujan identities shortly after he encountered them while working on his PhD in statistical mechanics about 20 years ago.

Although originally discovered by L. J. Rogers in 1894, the identities became famous through the work of Ramanujan, who was largely self-

taught and worked instinctively.

In 1913, Ramanujan sent a letter from his native India to the British mathematician G. H. Hardy that included the two identities that Rogers discovered and a third formula that showed these identities are essentially modular functions and their quotient has the special property that its singular values are algebraic integral units. That result came to be known as the Rogers-Ramanujan continued fraction.

Hardy was astonished when he saw the formulas. "I had never seen anything in the least like this before," Hardy wrote. "A single look at them is enough to show they could only be written down by a mathematician of the highest class. They must be true because no one would have the imagination to invent them."

"Ramanujan seemed to produce this result out of thin air," Ono says.

Ramanujan died in 1920 before he could explain how he conjured up the formulas. "They have been cited hundreds of times by mathematicians," Ono says. "They are used in statistical mathematics, conformal field theory and number theory. And yet no one knew whether Ramanujan just stumbled onto the power of these two identities or whether they were fragments of a larger theory."

For nearly a century, many great mathematicians have worked on solving the mystery of where Ramanujan's formulas came from and why they should be true.

Ono uses the analogy of going for a walk in a creek bed and discovering a piece of gold. Had Ramanujan accidentally found a random nugget? Or was he drawn to that area because he knew of a rich seam of gold nearby?

Warnaar was among those who pondered these questions. "Just like digging for gold, in mathematics it's not always obvious where to look for a solution," he says. "It takes time and effort, with no guarantee of success, but it helps if you develop a lot of intuition about where to look."

Finally, after 15 years of focusing almost entirely on the Rogers-Ramanujan identities, Warnaar found a way to embed them into a much larger class of similar identities using something known as representation theory.

"Ole found the mother lode of identities," Ono says.

When Ono saw Warnaar's work posted last November on arXiv.org, a mathematics-physics archive, his eyes lit up.

"It just clicked," Ono recalls. "Ole found this huge vein of gold, and we then figured out a way to mine the gold. We went to work and showed how to come full circle and make use of the formulas. Now we can extract infinitely many functions whose values are these beautiful algebraic numbers."

"Historically, the Rogers-Ramanujan identities have tantalized mathematicians," says George Andrews, a mathematician at Penn State and another top authority on the identities. "They have played an almost magical role in many areas of math, statistical mechanics and physics."

The collaboration of Warnaar, Ono and Griffin "has given us a big picture of the general setting for these identities, and deepened our theoretical understanding for many of the breakthroughs in this area of mathematics during the past 100 years," Andrews says. "They've given us a whole new set of tools to be able to attack new problems."

"It's incredibly exciting to solve any problem related to Ramanujan, he's such an important figure in mathematics," Warnaar says. "Now we can move on to more questions that we don't understand. Math is limitless, and that's fantastic."

More information: <http://arxiv.org/abs/1401.7718>

Provided by Emory University

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