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Researcher finds solution to problem in 19th-century theory of meromorphic functions

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THEOREM 1.2. *Let f be a transcendental meromorphic function in the complex plane. Let $k \geqslant 2$ be an integer, and let $\varepsilon > 0$. Let $A \subset \mathbb{C}$ be a finite set of complex numbers. Then we have*

$$(k-1)\overline{N}(r,\infty,f) + \sum_{a\in A} N_1(r,a,f) \leqslant N(r,0,f^{(k)}) + \varepsilon T(r,f) \qquad (1.4)$$

for all $r > e$ outside a set $E \subset (e,\infty)$ of logarithmic density 0. Here E depends on f, k, ε and A.

The statement of Gol’dberg conjecture from the original paper.

General theory of meromorphic functions in the complex plane began in the nineteenth century, when E. Picard proved his famous 'Picard's little theorem'. Then, in the 1920s, R. Nevanlinna created the modern theory of meromorphic functions, where his 'second main theorem (SMT)' provides a far-reaching generalization of Picard's theorem. Nowadays, the theory is well-established as a result of many excellent research studies. Nevertheless, the theory still has several unresolved problems, including the following one:

Conjecture of Gol'dberg, middle 1980-s: For every transcendental meromorphic function in the plane, the frequency of distinct poles is governed by the frequency of zeros of the second derivative.

Now, Tokyo Tech mathematician Katsutoshi Yamanoi has solved this conjecture.

The solution is based on two important developments in Nevanlinna theory, which are interesting for their own sake. The first one is a generalization of SMT for small moving targets. The other is a reversion of SMT.

The proof shows that Gol'dberg's conjecture is true in more general form. The results described by Yamanoi in this paper are an important breakthrough in the theory of meromorphic functions.

More information: Yamanoi, K. Zeros of higher derivatives of meromorphic functions in the complex plane, *Proc. London Math. Soc.* (2012), 78 pages. DOI:10.1112/plms/pds051

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