Can math models of gaming strategies be used to detect terrorism networks?

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The answer is yes, according to a paper in the *SIAM Journal on Discrete Mathematics*. In a paper published in the journal last month, authors Anthony Bonato, Dieter Mitsche, and Pawel Pralat describe a mathematical model to disrupt flow of information in a complex real-world network, such as a terrorist organization, using minimal resources.

Terror networks are comparable in their structure to hierarchical organization in companies and certain online social networks, where information flows in one direction from a source, which produces the information or data, downwards to sinks, which consume it. Such networks are called hierarchical social networks.

"In such networks, the flow of information is often one way," explains author Pawel Pralat. "For example, a celebrity such as Justin Bieber sends out a tweet, which is sent to millions of his followers. These followers send out their own retweets, and so on. We may therefore view hierarchical social networks as directed networks without cycles, or directed acyclic graphs (DAGs)."

Here, there is no requirement for reciprocity (the celebrity does not necessarily follow his or her followers). Similarly, in a terrorist network, the leaders pass plans down to the foot soldiers, and usually only one messenger needs to receive the message for the plan to be executed.

Disruption of the flow of information would correspond to halting the spread of news in an online social network or intercepting messages in a terror network.

The authors propose a generalized stochastic model for the flow and disruption of information based on a two-player outdoor game called "Seepage," where players who depict agents attempt to block the movement of another player, an intruder, from a source node to a sink. "The game—motivated by the 1973 eruption of the Eldfell volcano in Iceland—displays some similarities to an approach used in mathematical counterterrorism, where special kinds of DAGs are used to model the disruption of terrorist cells," says Pralat.

The motivating eruption caused a major crisis at the time, as lava flow threatened to close off the harbor, the island's main source of income. In the game, inhabitants attempt to protect the harbor by pouring water on the volcanic lava to halt its progress. A mathematical model of the game pits two opponents against each other—the sludge, or intruder, against the greens, or agents— forming a directed acyclic graph, with one source (the top of the volcano) and many sinks representing the lake. The parameter, "seepage," represents the amount of contamination, and the "green number" corresponds to the number of agents required to halt it.

A previous study modeled terrorist cells as partially ordered sets (a special kind of DAG), which are often used in mathematics to analyze an ordering, sequencing, or arrangement of distinct objects. In such a system, terrorist plans are formulated by nodes at the top of the hierarchy, which represent the leaders or maximal nodes of the set. The plans are transmitted down to the nodes at the bottom: these represent foot soldiers in a terror network or minimal nodes in the set who would be presumed to carry out these plans. The assumption is that one messenger is sufficient for reception and execution of the plan. Thus, if the partially ordered set represents a courier network for a terrorist organization, the intention would be to block all routes from the maximal node to the minimal nodes by capturing or killing a subset of agents.

In this paper, the authors utilize the similarities in the previous terrorist cell model to Seepage, where greens try to prevent the sludge from moving to the sinks by blocking nodes. A number of different winning strategies employed by both players are explored when played on a DAG. The seepage and green number for disrupting a given hierarchical social network are analyzed.

The primary difference from the previous study's model is that the Seepage model is dynamic: greens can move and choose new sets of nodes over time. The authors determine that Seepage is a more realistic model of counterterrorism, as the agents do not necessarily act all at once, but over time.

The analysis is made in two types of terrorist network structures, as Pralat explains, "We consider two extreme profiles: one where the network is regular, where every agent has about the same number of connections. The second profile is power law, where some agents have many connections, but most have very few." This is analyzed by considering the total degree distribution of nodes in the DAG. In regular DAGs, each level of the DAG would have nodes with about the same out-degree (number of outgoing edges emanating from a node), while power law DAGs would have many more low-degree nodes and a few with high degrees.

Mathematical analysis allows the authors to determine what point in a network would be most effective for disrupting messages. "Our mathematical results reinforce the view that intercepting the information or message in a hierarchical social network following a power law is more difficult close to levels near the source. For regular networks, it does not matter as much where the message is disrupted," says Pralat. "Future work could look at more complex profiles of networks, along with developing effective algorithms for disrupting the flow of information in a DAG using our game-theoretic approach."

More information: Vertex-Pursuit in Random Directed Acyclic Graphs, Anthony Bonato, Dieter Mitsche and Pawel Pralat, *SIAM Journal on Discrete Mathematics*, 27(2), 732–756. (Online publish date: April 16, 2013). epubs.siam.org/doi/abs/10.1137/120866932

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