Gravity's lingua franca: Unifying general relativity and quantum theory through spectral geometry

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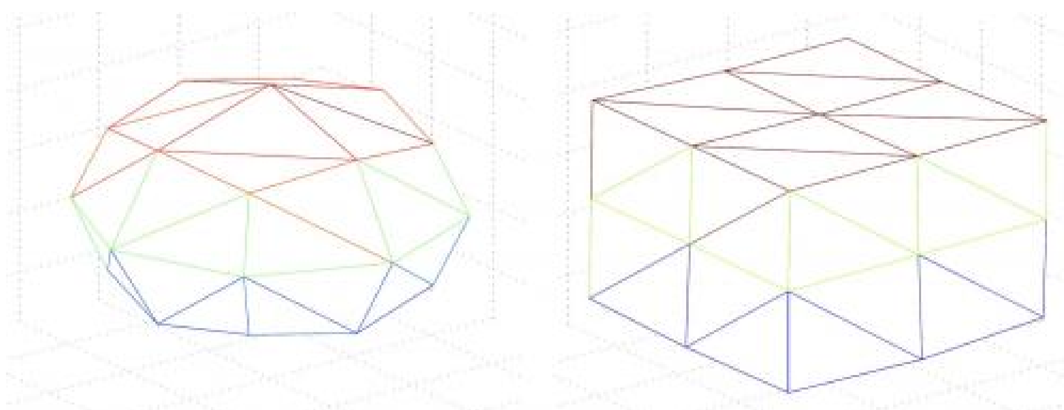

Snapshots of the algorithm starting with a sphere and finding the cube from its spectrum alone. Courtesy and acknowledgement Achim Kempf. Copyright 2013 by The American Physical Society.

(Phys.org) —Mathematics is, in essence, an artificial language for precisely articulating theories about the physical world. Unlike natural language, however, translating different classes of mathematics can be difficult at best. Such is the case encountered in the attempt to unify general relativity and quantum theory, since they are expressed in differential geometry and functional analysis, respectively. That being said, spectral geometry – a field in mathematics which concerns relationships between geometric structures of manifolds and spectra of canonically defined differential operators – may resolve this longstanding quandary by allowing spacetime to be treated as simultaneously continuous and discrete, essentially relating the frequency-based *ringing* of the fabric of spacetime to its manifold-based *shape*. Recently, scientists at California Institute of Technology, Princeton University, University of Waterloo, and University of Queensland normalized and segmented spectral geometry into small, finite-dimensional steps. They then demonstrated their approach of calculating the shapes of two-dimensional objects from their vibrational spectra as being viable in two, and possibly more, dimensions.

Prof. Achim Kempf discussed the research he, David Aasen, Tejal Bhamre conducted. "Before the new results," Kempf tells Phys.org, "it was thought that spectral geometry is too nonlinear – and therefore simply too hard to use – for the purpose of unifying general relativity and quantum theory. In the new paper, however, we showed that spectral geometry can be tamed and made into a very useful practical method, namely by suitably cutting it into small linear, and therefore manageable, pieces."

Kempf notes that in special cases, spectral geometry has certain ambiguities: mathematically, such as special curved shapes in high dimensions that have the same spectrum – that is, they would *sound* the same of we could detect higher dimensions. "The worry has been that if there were too many such ambiguities also in our three-dimensional world, this could make spectral geometric impractical as a tool in physics," Kempf explains. "In the new paper we showed that, fortunately, the small linearized steps that we take are almost always ambiguity-free – and for two dimensional shapes in three dimensions, we didn't find ambiguities at all. Relatedly, it would be very interesting to extend spectral geometry from a description of space *at* each time to a unified description of both space *and* time. This still needs to be developed further."

That being said, Kempf points out that their idea – addressing spectral geometry's difficulty and ambiguities by regularizing and segmenting spectral geometry into finite-dimensional steps – works very well. "The computation time can be a little long," he notes, but we think that we will be able to significantly speed up the calculations. We'd like to be able to run them, for example, on a smartphone."

A single key insight enabled the researchers addressed these challenges in two ways. Essentially, as far as the mathematics is concerned, the problem was to find a method that would allow one to calculate the shape of an object from the sound that it makes when vibrating. "To this end, the key insight was that this spectral geometric problem, in spite of being highly nonlinear, can actually be tamed with our strategy, which has two components," Kempf explains. "First, make the nonlinear calculations manageable by cutting them into small doable steps." In practice, he notes that the computer does this by starting with some random shape, such as the shape of a sphere. Then, while it keeps comparing the sound of the sphere to the sound of the object that it needs to identify, the computer will change the shape of the sphere until it reaches the shape of the object that it had to identify.

"The second step is to regularize – that is, don't try to get all of the shape's details at once," Kempf says. "Instead, calculate the rough shape from just part of the sound spectrum." By then incrementally using more of the sound spectrum, this approach allows them to specify the shape with increasing accuracy.

"The beauty of our new spectral geometry is that it allows us to describe the shape of a vase, or eventually the shape of the fabric of spacetime, through so-called invariants – that is, by quantities that do not depend on any choice of coordinate system," Kempf adds. "This is important because if we're to develop a theory that unifies quantum theory and general relativity, key quantities fundamentally cannot depend on manmade choices, such as which coordinate system one wants to use."

Kempf then summarized the relation of their approach, which offers a gauge-independent identification of the metric's degrees of freedom in terms of invariants that should be ready to quantize, with several other mathematical attempts to unify general relativity and quantum theory.

- Loop quantum gravity and string theory: "The new spectral geometric methods are deeply related to generalized Heisenberg uncertainty principles – and in fact, the new work grew out of studies of such principles, which have been shown to be related to loop quantum gravity as well as to string theory by myself and in collaboration with Martin Bojowald[1]."
- Causal sets: "Perhaps, but it's not clear if there's a connection."
- Garrett Lisi's E8 proposal: "Probably no connection."
- Noncommutative geometry: "Alain Connes' program of noncommutative geometry shows that curved spaces can be described by a spectral triple, which includes the spectrum of the Dirac operator. It's not clear if the spectrum of the Dirac operator alone is sufficient to calculate the shape of a curved space. The new spectral geometric methods that we present here can be used to explore this interesting question further, and in fact we're working on this."
- Supergravity: "Our new results apply to gravity and do not require supersymmetry. This is good because there's still no solid evidence that supersymmetry exists in nature."
- Twistor models: "No connection known."

Moving forward, says Kempf, the scientists are working on generalizing the new methods to shapes that are curved in both space and time, since that will then be useful for addressing some of the key questions of cosmology – including the question of how it all began." More specifically, Kempf adds that while quantum fluctuations are today almost immeasurably small, it's thought that spacetime itself arose from a kind of quantum jump. "Our results bring us a step closer to being able to explicitly calculate the quantum ringing of spacetime, which could then tell us more about the origin of our universe."

In terms of other areas of research that might benefit from their study, Kempf points out that experimentalists still have a long way to go to measure quantum gravity effects directly. "However," he adds, "our new methods can also be used to program a computer to calculate the shape of objects from their sound. Moreover," he concludes, "we're planning to improve our algorithm to make it much faster. This could open up engineering applications, for example, by allowing machines to quickly identify shapes from a simple spectral fingerprint."

More information: Shape from Sound: Toward New Tools for Quantum Gravity. *Physical Review Letters* 110, 121301 (2013), doi:10.1103/PhysRevLett.110.121301

Related:
[1]Generalized uncertainty principles and localization of a particle in discrete space. *Physical Review D* 86, 085017 (2012), DOI:10.1103/PhysRevD.86.085017
Spacetime could be simultaneously continuous and discrete, in the same way that information can be. *New Journal of Physics* 12 115001 (2010), doi:10.1088/1367-2630/12/11/115001