A mathematical formula to decipher the geometry of surfaces like that of cauliflower

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Scientists at the Universidad Carlos III of Madrid (UC3M) have taken part in a research project that describes, for the first time, that laws that govern the development of certain complex natural patterns, such as those found on the surface of cauliflower.

The scientists have found a formula that describes how the patterns found in a multitude of natural structures are formed. "We have found a model that describes, in detail, the evolution in time and in space of cauliflower-type fractal morphologies for nanoscopic systems", explains Professor Rodolfo Cuerno, of UC3M's Mathematics Department, author of the research, together with scientists from Universidad Pontificia Comillas (UPCO), the Instituto de Ciencia de los Materiales de Madrid (the Materials Science Institute of Madrid) of the Consejo Superior de

Investigaciones Científicas (CSIC) (Spanish National Research Council), la Escuela Politécnica de París (Polytechnic School of Paris, France) and the Universidad Católica de Lovaina (Catholic University of Louvain, Belgium).

This work, which was recently published in the New Journal of Physics, falls within the field of fractal geometry, which is based on the mathematical description of many natural forms, such as sea coasts, the borders between countries, clouds, snowflakes and even the networks of blood vessels. A fractal is characterized because its parts are similar to the whole. "In the case of cauliflowers, this property (self-similarity) becomes evident if you look closely at a photo of them," says another of the researchers, Mario Castro, a professor at UPCO. "In fact," he adds, "without more information, it is impossible to know the size of the object." This way, using relatively simple algorithms, complex structures almost indistinguishable from certain landscapes, leaves or trees, for example, can now be generated. "However, the general mechanisms that govern the appearance or evolution over time of those natural structures have rarely been identified beyond a merely visual or geometric reproduction," clarifies the researcher.

From the supermarket to the laboratory

The cauliflower-type morphologies were known is this realm in an empirical way, but no one had provided a model like the one that these scientists have developed. "In our case," they comment, "the connection came about naturally when a certain ingredient (noise) was added to a related model that we had worked on previously. When we did that, in the numeric simulations, surfaces appeared, and we quickly identified them as the ones that our experiment colleagues had been able to obtain, under the right conditions, in their laboratories." Based on the characteristics of this theoretical model, they have inferred general mechanisms that can be common and can help in making models of

other very different systems, such as a combustion front or a cauliflower like the ones that can be found in any supermarket.

Fractals of this type are interesting because they are ubiquitous, that is, they appear in systems that vary widely in their nature and dimensions. In general, fractals can be found in any branch of the natural sciences: mathematics (specific types of functions), geology (river basins or the outline of a coast), biology (forms of aggregate cells, of plants, of the network of blood vessels...), physics (the growth of amorphous solid crystals or the distribution of galaxies), chemistry (the distribution in space of the reagents of chemical reactions). Moreover, they have also been studied due to their relationship with structures created by man, such as communication and transportation networks, city layouts, etc.

This finding may help to discover concrete applications for improving the technologies used in thin film coatings, and to understand the conditions under which they are smooth or have wrinkles or roughness. "This is also useful in generating textures in computer simulations," the researchers point out. "And, conceptually," they add, "this can give us clues about the general mechanisms involved in forming structures in areas that are very different from the ones in which the model was formulated, such as those in which there is competition for growth resources among the various parts of the system."

More information: Universality of cauliflower-like fronts: from nanoscale thin films to macroscopic plants, Authors: Mario Castro, Rodolfo Cuerno, Matteo Nicoli, Luis Vázquez and Josephus G. Buijnsters, Journal: *New J. Phys.* 14 (2012) 103039
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