Math detects contamination in water distribution networks

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None of us want to experience events like the Camelford water pollution incident in Cornwall, England, in the late eighties, or more recently, the Crestwood, Illinois, water contamination episode in 2009 where accidental pollution of drinking water led to heart-wrenching consequences to consumers, including brain damage, high cancer risk, and even death. In the case of such catastrophes, it is important to have a method to identify and curtail contaminations immediately to minimize impact on the public.

A paper published earlier this month in the *SIAM Journal on* Applied Mathematics considers the identification of contaminants in a water distribution network as an optimal control problem within a networked system.

"Water supply networks are an essential part of our infrastructure. Sometimes the water in such a network can be contaminated, often by human error, causing the use of polluted water for drinking water production. In the case of such a situation, it is important to have a method to identify the location of the pollution source," says the paper's author, Martin Gugat, explaining the significance of his work.

The paper considers a water distribution network with a finite number of nodes where contamination can occur in the pipes.

"The contamination spreads dynamically through the network with time. So, in order to model the system, a model of the evolution in time is

necessary," explains Gugat. "In our approach, we use a partial differential equation (PDE) to model how pollution spreads in the network."

By using a PDE model for transport of contaminants, the problem of identifying the source becomes an optimal control problem. The solution is calculated using equidistant time grids, which allows one to determine the values of contamination at all potential sources on the time grid. Available data on pollution and network flow is incorporated into the model.

Employing certain assumptions for travel times through the pipes, the author uses a least-squares method to solve the problem. The least squares method provides approximate solutions to optimization problems that are relatively efficient to compute using the tools of numerical linear algebra.

This provides a fast method to identify possible contamination sources, explains Gugat. "For a really accurate model, however, a full system of three-dimensional PDEs is necessary. But with three-dimensional PDEs, simulation is only possible for small networks," he says. "This illustrates that to solve real life problems on real networks, there is a trade-off between the accuracy of the model and its utility."

While the method is tested numerically in the paper, additional work would involve testing the system with an existing water network to demonstrate its workability in practice.

Another future direction is toward elimination of the contaminant. "The second step after the identification of the contamination source is a strategy to flush the polluted water out of the network as fast as possible with acceptable operational cost. The development of an optimal strategy for such a rehabilitation of the water supply is an interesting question for

future research," says Gugat.

"For a more detailed model of the process, more complex nonlinear PDEs could be used," he continues. "The cost of the numerical treatment of complex PDEs for large networks is prohibitive. Applied mathematics has to offer models that can be used according to the problem requirements to solve problems with network graphs of a realistic size."

More information: "Contamination Source Determination in Water Distribution Networks" Martin Gugat, *SIAM Journal on Applied Mathematics*, 72(6), 1772. (Online publish date: 5 November 2012) epubs.siam.org/doi/abs/10.1137/110859269

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