Advances in mathematical description of motion

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Complex mathematical investigation of problems relevant to classical and quantum mechanics by EU-funded researchers has led to insight regarding instabilities of dynamic systems. This is important for descriptions of various phenomena including planetary and stellar evolution.

Well known Euclidean geometry is used to measure one-dimensional (1D) quantities such as length or angle. Symplectic geometry is used to describe even-dimensional (such as 2D, 4D and 6D) objects.

The concept of symplectic structure arose from the study of classical mechanical systems such as planets orbiting the sun, oscillating pendulums and Newton's falling apple. The trajectory of such systems is well defined if one knows two pieces of information, position and velocity (or, more precisely, momentum).

Quantum physics and Heisenberg's Uncertainty Principle led to a modification of mathematics. Following the above example, a particle could no longer be considered as occupying a single point but rather as lying in a region of space, defined by two position coordinates and two velocity coordinates (four dimensions).

Continued evolution of mathematical theories led to the use of so-called Lie groups for the study of symmetries of geometric structures because it enabled 'reducing' the geometric space (dimension) under study to a much smaller one without sacrificing accuracy and clarity.

However, the observation of mathematical singularities, or points at which a given mathematical object is undefined or fails to be mathematically 'well-behaved', resulted in important effects on the dynamical stability of mechanical systems as well as instability of mathematical solutions.

The complex mathematics defining mechanical and dynamical systems was the focus of the 'Hamiltonian actions and their singularities' (Hamacsis) project, given that Hamiltonian equations provide a way of connecting classical mechanics with quantum mechanics.

Among the many insights gained by Hamacsis, extensive investigation of so-called cotangent bundles of vectors provided important descriptions of their reduced spaces in the cases of symmetric actions and singularities.

In addition, researchers explicitly showed how singularities of symmetric actions in several types of mechanical and dynamical systems affect stability properties. This is applicable to physically observed steady motions such as that in uniformly rotating bodies and planetary and stellar evolution.

The complex and innovative mathematics behind the Hamacsis project resulted in numerous publications in peer reviewed scientific journals. Outcomes significantly enhance our understanding of classical and quantum mechanics related to the motion of complex dynamical systems .

Provided by CORDIS

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