Combining centuries-old mathematical theorems provides efficient approach for characterizing nanoparticles' shape

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(PhysOrg.com) -- Gregg Gallatin, a researcher at the NIST Center for Nanoscale Science and Technology, has shown that combining a nineteenth century flux theorem with an eighteenth century mathematical operation provides a convenient technique for using scattered light to count nanoparticles and to characterize their shapes.

This technique is useful both for determining how a given distribution of nanoparticle shapes affects the properties of nanoparticle functionalized materials as well as for categorizing how biological systems incorporate nanoparticles of different shapes.

The mathematical approach, which combines Gauss's Law with Fourier transforms, can also be used as a starting point to solve a wide variety of standard problems in mathematics and physics beyond nanotechnology. Because of the ubiquity of digital data derived from Fourier transforms, the approach is likely to find broad application to a range of physical science and engineering measurements.

Using the technique, Gallatin demonstrates how Porod's law, which describes how x-rays scatter from small spherically-shaped particles, can be re-derived and extended to the broader case of particles that are nonspherical, thereby providing a powerful and useful approach for determining the shape of nanoparticles using x-ray scattering. He then demonstrates that this approach can be further extended to visible light scattering, which depends on the moments of the nanoparticle shape and therefore provides a more general method for measuring nanoparticle shape from scattering data.

The technique of combining Gauss's Law with Fourier transforms can also be applied to the classical physics problem of Fraunhofer diffraction, providing an explicit formula for the diffraction pattern of arbitrary polygonal-shaped openings in an opaque screen in terms of the vertices of the polygon. It is also applicable to a variety of mathematics problems, including the Hopf Umlaufsatz, which states that the angle of the tangent along a simple smooth closed curve turns by 360 degrees when making a complete circuit around the curve; Stokes' Law, which relates integrals over an area in two dimensions to the one dimensional curve bounding the area; and the isoperimetric inequality, which states that a circle is the shape that encloses the largest area for a given circumference.

Given the simplicity and generality of this mathematical technique, Gallatin believes that it can be applied to many other problems as well.

More information: Fourier, Gauss, Fraunhofer, Porod and the shape from moments problem, G. M. Gallatin, Journal of Mathematical Physics 53, 013509-013509-13 (2012).

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