Mathematically detecting bubbles before they burst

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From the dotcom bust in the late nineties to the housing crash in the run-up to the 2008 crisis, financial bubbles have been a topic of major concern. Identifying bubbles is important in order to prevent collapses that can severely impact nations and economies.

A paper published this month in the *SIAM Journal on Financial Mathematics* addresses just this issue. Opening fittingly with a quote from New York Federal Reserve President William Dudley emphasizing the importance of developing tools to identify and address bubbles in real time, authors Robert Jarrow, Younes Kchia, and Philip Protter propose a mathematical model to detect financial bubbles.

A financial bubble occurs when prices for assets, such as stocks, rise far above their actual value. Such an economic cycle is usually characterized by rapid expansion followed by a contraction, or sharp decline in prices.

"It has been hard not to notice that financial bubbles play an important role in our economy, and speculation as to whether a given risky asset is undergoing bubble pricing has approached the level of an armchair sport. But bubbles can have real and often negative consequences," explains Protter, who has spent many years studying and analyzing financial markets.

"The ability to tell when an asset is or is not in a bubble could have important ramifications in the regulation of the capital reserves of banks as well as for individual investors and retirement funds holding assets for the long term. For banks, if their capital reserve holdings include large investments with unrealistic values due to bubbles, a shock to the bank could occur when the bubbles burst, potentially causing a run on the bank, as infamously happened with Lehman Brothers, and is currently happening with Dexia, a major European bank," he goes on to explain, citing the significance of such inflated prices.

Using sophisticated mathematical methods, Protter and his co-authors answer the question of whether the price increase of a particular asset represents a bubble in real time. "[In this paper] we show that by using tick data and some statistical techniques, one is able to tell with a large degree of certainty, whether or not a given financial asset (or group of assets) is undergoing bubble pricing," says Protter.

This question is answered by estimating an asset's price volatility, which is stochastic or randomly determined. The authors define an asset's price process in terms of a standard stochastic differential equation, which is driven by Brownian motion. Brownian motion, based on a natural process involving the erratic, random movement of small particles suspended in gas or liquid, has been widely used in mathematical finance. The concept is specifically used to model instances where previous change in the value of a variable is unrelated to past changes.

The key characteristic in determining a bubble is the volatility of an asset's price, which, in the case of bubbles is very high. The authors estimate the volatility by applying state of the art estimators to real-time tick price data for a given stock. They then obtain the best possible extension of this data for large values using a technique called Reproducing Kernel Hilbert Spaces (RKHS), which is a widely used method for statistical learning.

"First, one uses tick price data to estimate the volatility of the asset in question for various levels of the asset's price," Protter explains.

"Then, a special technique (RKHS with an optimization addition) is employed to extrapolate this estimated volatility function to large values for the asset's price, where this information is not (and cannot be) available from tick data. Using this extrapolation, one can check the rate of increase of the volatility function as the asset price gets arbitrarily large. Whether or not there is a bubble depends on how fast this increase occurs (its asymptotic rate of increase)."

If it does not increase fast enough, there is no bubble within the model's framework.

The authors test their methodology by applying the model to several stocks from the dot-com bubble of the nineties. They find fairly successful rates in their predictions, with higher accuracies in cases where market volatilities can be modeled more efficiently. This helps establish the strengths and weaknesses of the method.

The authors have also used the model to test more recent price increases to detect bubbles. "We have found, for example, that the IPO [initial public offering] of LinkedIn underwent bubble pricing at its debut, and that the recent rise in gold prices was not a bubble, according to our models," Protter says.

It is encouraging to see that mathematical analysis can play a role in the diagnosis and detection of bubbles, which have significantly impacted economic upheavals in the past few decades.

More information: How to Detect an Asset Bubble Robert Jarrow, Younes Kchia, and Philip Protter, *SIAM Journal on Financial Mathematics* 2 (2011), pp 839-865 (Online publish date: October 12, 2011)

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