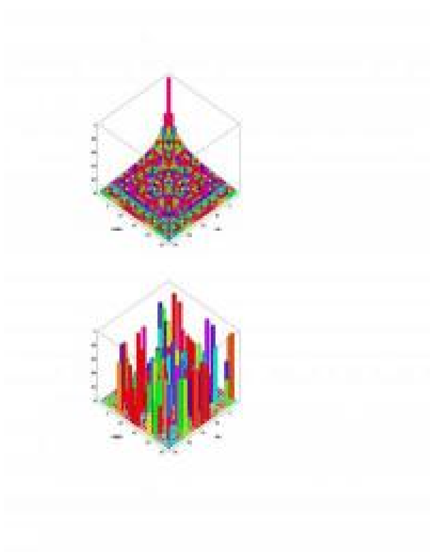
Experimental mathematics: Computing power leads to insights

October 13 2011



Mathematicians often work with matrices, which are arrays of numbers. When written on a page, a matrix can look like a sea of numbers, so any patterns that might occur in the numbers can be difficult to discern. More and more, mathematicians are turning to graphical representations of matrices, like the two examples here. By using color and form to indicate the values of the numbers in the matrix, these graphical representations can instantly give a sense of the patterns in the matrix. The first picture is a representation of a matrix in which the numbers exhibit a clear pattern; the second picture, by contrast, is a matrix in which the numbers are random. Graphic by David Bailey and Jonathan Borwein. Request their permission before reproducing the graphic.

In his 1989 book "The Emperor's New Mind", Roger Penrose commented on the limitations on human knowledge with a striking example: He conjectured that we would most likely never know whether a string of 10 consecutive 7s appears in the digital expansion of the number pi. Just 8 years later, Yasumasa Kanada used a computer to find exactly that string, starting at the 22869046249th digit of pi. Penrose was certainly not alone in his inability to foresee the tremendous power that computers would soon possess. Many mathematical phenomena that not so long ago seemed shrouded and unknowable, can now be brought into the light, with tremendous precision.

In their article "Exploratory Experimentation and Computation", to appear in the November 2011 issue of the *Notices of the American Mathematical Society*, David H. Bailey and Jonathan M. Borwein describe how modern computer technology has vastly expanded our ability to discover new mathematical results. "By computing mathematical expressions to very high precision, the computer can discover completely unexpected relationships and formulas," says Bailey.

Mathematics, the Science of Patterns

A common misperception is that mathematicians' work consists entirely of calculations. If that were true, computers would have replaced mathematicians long ago. What mathematicians actually do is to discover and investigate patterns---patterns that arise in numbers, in abstract shapes, in transformations between different mathematical objects, and so on. Studying such patterns requires subtle and sophisticated tools, and, until now, a computer was either too blunt an instrument, or insufficiently powerful, to be of much use in mathematics. But at the same time, the field of mathematics grew and deepened so much that today some questions appear to require additional capabilities beyond the human brain.

"There is a growing consensus that human minds are fundamentally not very good at mathematics, and must be trained," says Bailey. "Given this fact, the computer can be seen as a perfect complement to humans---we can intuit but not reliably calculate or manipulate; computers are not yet very good at intuition, but are great at calculations and manipulations."

Although mathematics is said to be a "deductive science", mathematicians have always used exploration, whether through calculations or pictures, to test ideas and gain intuition, in much the same way that researchers in inductive sciences carry out experiments. Today, this inductive aspect of mathematics has grown through the use of computers, which have vastly increased the amount and type of exploration that can be done. Computers are of course used to ease the burden of lengthy calculations, but they are also used for visualizing mathematical objects, discovering new relationships between such objects, and testing (and especially falsifying) conjectures. A mathematician might also use a computer to explore a result to see whether it is worthwhile to attempt a proof. If it is, then sometimes the computer can give hints about how the proof might proceed. Bailey and Borwein use the term "experimental mathematics" to describe these kinds of uses of the computer in mathematics.

Exploring Prime Numbers via Computers

Their article gives several examples of experimental mathematics; the computations of the digits of pi mentioned above is one of them. Another example is provided by computer explorations of a mathematical problem known as Giuga's Conjecture. This conjecture proposes that, for any positive integer n, one can check definitively whether n is prime by calculating a certain sum in which n appears in the exponent of the summands. That sum would have a certain value, call it S, if and only if n is prime; stated differently, that sum would not have the value S if and only if n is composite. Although the conjecture dates to 1950, it has never been proved and seems out of reach by conventional mathematical methods.

However, Bailey and Borwein, along with their collaborators, were able to use computers to show that any number that is an exception to Giuga's Conjecture must have more than 3,678 distinct prime factors and be more than 17,168 decimal digits long. That is, any shorter composite number cannot result in the value S. This does not prove Giuga's Conjecture is true, but it is a compelling piece of evidence in favor of the conjecture's truth. This kind of empirical evidence is sometimes just what is needed to generate enough confidence for a mathematician to dedicate energy to seeking a full proof. Without such confidence, the inspiration to push through to a proof might not be there.

Impact on Education

In addition to discussing state-of-the-art uses of computers in mathematics, the article also touches on the need to refashion mathematics education to give students the tools of experimental mathematics. "The students of today live, as we do, in an information-rich, judgment-poor world in which the explosion of information, and of tools, is not going to diminish," says Borwein. "So we have to teach judgment (not just concern with plagiarism) when it comes to using what is already possible digitally. Additionally, it seems to me critical that we mesh our software design---and our teaching style more generally---with our growing understanding of our cognitive strengths and limitations as a species."

Provided by American Mathematical Society

Citation: Experimental mathematics: Computing power leads to insights (2011, October 13) retrieved 24 April 2024 from
https://phys.org/news/2011-10-experimental-mathematics-power-insights.html