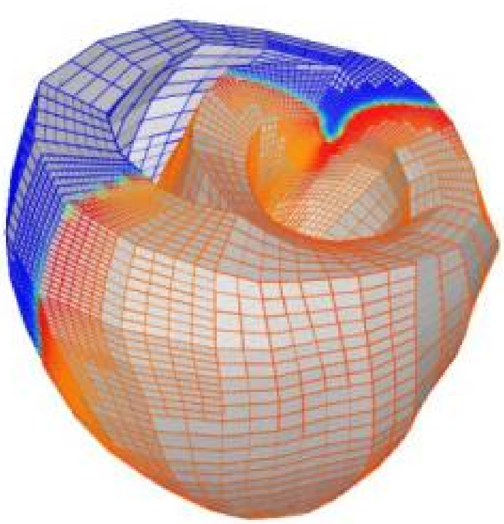
PHYS ORG

Taking mathematics to heart

March 14 2011



This picture is from research of Wenjun Ying of the Department of Mathematics at Shanghai Jiao Tong University, who studies electrical wave propagation in the heart with the technique of adaptive mesh refinement. The picture shows the adaptively refined grids that keep track of the propagating electrical waves in a virtual dog ventricle. The region covered by the blue grids denotes the area where the cardiac tissue is not activated while the red area shows where the tissue is activated. Credit: Wenjun Ying, Department of Mathematics, Shanghai Jiao Tong University

Did you know that heart attacks can give you mathematics? That statement appears on the web site of James Keener, who works in the mathematics of cardiology. This area has many problems that are ripe for unified attack by mathematicians, clinicians, and biomedical engineers. In an article to appear in the April 2011 issue of the Notices of the American Mathematical Society, John W. Cain, a mathematician at Virginia Commonwealth University, presents a survey of six ongoing Challenge Problems in mathematical cardiology. Cain's article emphasizes cardiac electrophysiology, because some of the most exciting research problems in mathematical cardiology involve electrical wave propagation in heart tissue.

At some point in our lives, many of us will undergo an electrocardiogram (ECG), a recording of electrical activity in the heart. To understand where these tiny electrical currents originate, we must zoom in to the molecular level. Bodily fluids, such as blood, contain positively charged ions. When these ions traverse cell membranes, they cause electrical currents, which in turn elicit changes in the voltage V across the membrane. If a sufficiently strong stimulus current is applied to a sufficiently well-rested cell, then the cell experiences an "action potential": V suddenly spikes and remains elevated for a prolonged interval. These action potentials govern heartbeat patterns and are therefore critical to understanding and treating disorders like arrhythmia (abnormal heart rhythms) and in particular tachycardia (faster than normal heart rhythms).

Taking the Nobel Prize-winning work of Hodgkin and Huxley as a starting point, researchers have created mathematical models of the cardiac action potential by viewing the cardiac cell membrane as an electrical circuit. A major challenge that Cain identifies is striking a balance between feasiblity and complexity: Minimize complications in the model, so that it is amenable to mathematical analysis, but add sufficient detail, so that the model reproduces as much clinically relevant data as possible. The equations that govern the model---nonlinear partial differential equations---cannot be solved explicitly, and solutions must be obtained through approximation by numerical methods. Adding further complications are the intricate geometry of the heart, with its four chambers and connections to veins and arteries, and the fact that different types of cardiac tissue have different conduction properties.

Cain goes on to discuss various cardiac phenomena and the mathematics that can be used to describe them. One example is heart rhythm: The regular, coordinated contraction of the heart muscle that pumps blood through the body. Improving the understanding and treatment of irregularities in that rhythm is critical in the fight against heart disease.

A healthy heart does not beat in a perfectly regular pattern; in fact, such a pattern would be a sign of potentially serious pathologies. The body's autonomic nervous system uses neurotransmitters to speed up or slow down the heart, and tiny fluctuations in those substances induce variability in the intervals between consecutive beats. The RR interval is the interval between consecutive heartbeats measured in an ECG. Attempts to quantify heart rate variability (HRV) usually involve analyzing time series of RR intervals.

Unfortunately, some ways of analyzing RR time series give the same results for patients with healthy hearts and for those with fatal cardiac abnormalities. One challenge for mathematicians and statisticians is to devise quantitative methods for distinguishing between the RR time series of people with healthy hearts and the RR time series of those with cardiac pathologies. Cain asks, Can some pathologies be diagnosed solely by analysis of RR time series and, if so, which ones? To spot subtle pathologies, methods are needed for quantifying the "regularity" of a cardiac rhythm. Also, given the existing array of diagnostic tests that clinicians have at their disposal, there could be advantages in the use of "automated" mathematical/statistical methods.

More information: Cain's article, "Taking Math to Heart: Mathematical Challenges in Cardiac Electrophysiology", is freely available on the Notices web site - www.ams.org/notices.

Provided by American Mathematical Society