New fundamental limitation restricts position accuracy of quantum objects

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(PhysOrg.com) -- Although the uncertainty principle is probably the most well-known example of a fundamental limitation of measurement precision in quantum mechanics, it is not the only one. In fact, every physical system is characterized by a number of variables that do not change their values as the system evolves over time; such variables are called conserved quantities and they are said to obey a conservation law. The fact that some quantities cannot change their values suggests that there might be restrictions on the possible ways in which a measurement device can interact with a quantum object and extract information from it.

This expectation has been confirmed and made precise by the Wigner-Araki-Yanase (WAY) theorem, which was developed in the early 1960s. In a new study, researchers have extended this theorem by showing that the conservation of the total momentum of a quantum object and measuring apparatus places a fundamental limit on how accurately the object's position can be measured.

Professor Paul Busch and Ph.D. student Leon Loveridge, mathematical physicists at the University of York, UK, have published their study on this previously unknown fundamental limitation in a recent issue of *Physical Review Letters*.

The scope of the original WAY theorem was limited in that its proof only applied to a restricted class of physical variables and conserved quantities. Ever since the discovery of the theorem, researchers have wondered whether it might extend to the important case of the position of a quantum particle. In 1991, Japanese physicist Masanao Ozawa, then at Harvard University, developed a model which seemed to suggest that the position can be measured with arbitrary accuracy and repeatability using an interaction that leaves the total momentum of the quantum object and apparatus conserved.

In the new study, Busch and Loveridge have analyzed Ozawa's model and found that, contrary to Ozawa's conclusion, momentum conservation does in fact limit the accuracy and repeatability of position measurements: they have shown that good accuracy and repeatability of a position measurement can only be achieved by using a sufficiently large apparatus.

The researchers also developed an alternative model that identifies a particular condition underlying the WAY theorem: the so-called Yanase condition, which stipulates the compatibility of the indicator variable of the apparatus with the conserved quantity. This alternative model shows that, if it were allowed to disregard and violate the Yanase condition, position measurements could be done with arbitrary accuracy, even with a small apparatus. However, if one tries to exploit this escape route from the WAY theorem, one is only faced by the puzzling prospect of the same limitation reappearing for the apparatus indicator variable.

"This is perhaps surprising for a number of reasons," Loveridge told *PhysOrg.com*. "Firstly, it is exponentially more accurate than an old model of von Neumann which did not obey the conservation of momentum - one might have thought that by including the conservation law things should get worse. Secondly, in the discrete and bounded case one has to give up repeatability and the Yanase condition for accurate measurements with no size constraint. In this model, we can still have arbitrarily good accuracy and repeatability, without any constraint on the size."

As Busch and Loveridge explain, understanding these kinds of quantum limitations on measurements is important for developing a more complete description of physical reality. In addition to being of theoretical interest, such limitations must also be taken into account in the engineering of single quantum objects.

“On a fundamental level, it is important to understand any physical theory as thoroughly as possible, and in turn to understand how nature behaves or at least manifests itself through observation,” Busch said. “Some would say that this is the primary goal of scientific investigation. On a more practical level, as discussed in our paper, there are potential ramifications for the processing of (quantum) information in which the information is encoded in a continuous variable.”

More information: Paul Busch and Leon Loveridge. “Position Measurements Obeying Momentum Conservation.” *Physical Review Letters* 106, 110406 (2011). DOI:10.1103/PhysRevLett.106.110406