

Computer scientists make progress on math puzzle

October 28 2010

(PhysOrg.com) -- Two UT Dallas computer scientists have made progress on a nearly 4-decade-old mathematical puzzle, producing a proof that renowned Stanford computer scientist Don Knuth called "amazing" in his communication back to them.

Created by the mathematician John Conway and known as Topswops, the puzzle starts like this: Begin with a randomly ordered deck of cards numbered 1 to n , with n being however high a number you choose. Now count out the number of cards represented by whatever card is the top card, and turn that block of cards over on top of the remaining cards. Then count out the number of cards represented by the new top card and turn this whole block over on top of the remaining cards. Repeat until the card numbered 1 comes to the top (realizing that we know the card numbered 1 will always eventually come to the top).

Now here's what needs to be done: Calculate the maximum and minimum number of steps required with n number of cards.

Knuth had previously proved an exponential upper bound on the number of Topswops steps, and conjectured that one might also prove a matching lower bound. What Dr. Hal Sudborough and Dr. Linda Morales did, however, was to prove a lower bound that is much better than that proposed in Knuth's conjecture, and Knuth declared their proof technique both "elegant" and "amazing."

"What I find fascinating about a problem such as bounding the

Topswops function is connected to its simplicity, to its fundamental nature, and to the complexity and difficulty of finding an answer,” said Sudborough, the Founders Professor at the Erik Jonsson School of Engineering and Computer Science. “An easily described, easily communicated problem is invaluable for engaging a wide array of participants, from high school students to the most eminent mathematicians.”

He also cited Martin Gardner, a longtime columnist for *Scientific American*, who wrote of problems such as Topswops, “Let it not be supposed that those Conway card games are trivial. They deal with the theory of set permutations and not only may provide deep theorems but also may have a bearing on practical problems that arise in seemingly unrelated fields.”

And then there’s the sheer mathematical beauty that the problem reveals.

“The Topswops process is a simple one,” said Morales, a senior lecturer in computer science. “The basic algorithm is easily understood by almost anyone, regardless of their training or interests. But the simplicity is deceptive. Hiding behind it is a mathematical world of unexpected richness and beauty. Our research uncovered permutations whose iterate sequences have a fascinating structure, which upon analysis have revealed hitherto unknown lower bounds for the problem. There is much more to learn from the problem. We have tantalizing hints of more revelations just waiting to be uncovered.”

The lower bound result appears as “A Quadratic Lower Bound for Topswops” in the October 2010 issue of the journal *Theoretical Computer Science*, and is now included in Knuth’s *The Art of Computer Programming, Fascicle Two*, written as a precursor to the soon-to-appear *The Art of Computer Programming, Volume 4*.

Provided by University of Texas at Dallas

Citation: Computer scientists make progress on math puzzle (2010, October 28) retrieved 9 April 2024 from <https://phys.org/news/2010-10-scientists-math-puzzle.html>

This document is subject to copyright. Apart from any fair dealing for the purpose of private study or research, no part may be reproduced without the written permission. The content is provided for information purposes only.