Math goes to the movies

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All hairs are simulated in this hybrid volumetric/geometric collision algorithm developed by two of the authors, Aleka McAdams and Joseph Teran (together with additional co-authors). Credit: Copyright Disney Enterprises, Inc.

Whether it's an exploding fireball in "Star Wars: Episode 3", a swirling maelstrom in "Pirates of the Caribbean: At World's End", or beguiling rats turning out gourmet food in "Ratatouille", computer-generated effects have opened a whole new world of enchantment in cinema. All such effects are ultimately grounded in mathematics, which provides a critical translation from the physical world to computer simulations.

The use of mathematics in cinematic special effects is described in the article "Crashing Waves, Awesome Explosions, Turbulent Smoke, and Beyond: Applied Mathematics and Scientific Computing in the Visual Effects Industry", which will appear in the May 2010 issue of the *NOTICES OF THE AMS*. The article was written by three University of California, Los Angeles, mathematicians who have made significant contributions to research in this area: Aleka McAdams, Stanley Osher, and Joseph Teran.

Mathematics provides the language for expressing physical phenomena and their interactions, often in the form of partial differential equations. These equations are usually too complex to be solved exactly, so mathematicians have developed numerical methods and algorithms that can be implemented on computers to obtain approximate solutions. The kinds of approximations needed to, for example, simulate a firestorm, were in the past computationally intractable. With faster computing equipment and more-efficient architectures, such simulations are feasible today---and they drive many of the most spectacular feats in the visual effects industry.

Another motivation for development in this area of research is the need to provide a high level of controllability in the outcome of a simulation in order to fulfill the artistic vision of scenes. To this end, special effects simulation tools, while physically based, must be able to be dynamically controlled in an intuitive manner in order to ensure believability and the quality of the effect.

Penny's hair from BOLT is simulated using a clumping technique. Only a few hundred hairs were simulated, and the rest are added at render time. Image copyright Disney Enterprises, Inc. Credit: Copyright Disney Enterprises, Inc.

The area of computational fluid dynamics (CFD) provides many of the tools used in simulations of phenomena such as smoke, fire, and water. Before the use of CFD, computer-generated special effects such as explosions were driven by force fields applied to passive unconnected particles, a method that produced rather unrealistic results. Today, a combination of improved hardware and faster algorithms for CFD models have made such special effects much more realistic. CFD has also been used, unsurprisingly, to simulate water-based phenomena; in fact, such water simulation techniques were recognized by an Academy Award for Technical Achievement for the mathematician/computer scientist Ronald Fedkiw of Stanford University.

Mathematics also plays a key role in computer-generated animations of all kinds of solids, from animated characters to cityscapes. Virtually every computer-generated solid has an explicit mathematical representation as a meshed surface or volume. Flesh simulations can endow computer-generated characters with realistically bulging muscles and rippling fat. Hair simulation provides a realistic way to depict the highly complex phenomenon of thousands of hairs interacting and colliding. The article describes recent work by the the first and third authors that provides a new technique for hair simulation.

The effects industry is emerging as an exciting new frontier for mathematicians, one that uniquely combines mathematical insights with the art of moviemaking.

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