Rough calculations: New book lays out practical tools for educated guessing

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Graphic: Christine Daniloff

(PhysOrg.com) -- Time for some quick arithmetic: Is $3600 \times 4.4 \times 10^4 \times 32$ larger or smaller than 3×10^9?

Finding the right answer, says Sanjoy Mahajan, associate director for teaching initiatives at MIT's Teaching and Learning Laboratory, does not require crafting a long, tedious calculation. Instead, the key to solving this problem — and many others — lies in having informal tools on hand that let us attack the problem. Though the result may not be perfectly precise, he believes, intuitive mathematical reasoning is often sufficient for our needs.

“That’s not to say exact answers aren’t useful,” says Mahajan, “but if looking for them is your only approach, you may never get any answer at all. Sometimes it’s better to start with something rough.”

So while conventional math teaching is often a highly formal affair, with an emphasis on definitions, theorems, and proofs, Mahajan believes we should learn practical math tools and understand why they work. He outlines this philosophy — and explains those tools — in a new book, *Street-Fighting Mathematics: The Art of Educated Guessing and Opportunistic Problem-Solving*, being published this month by MIT Press.

Street-Fighting Mathematics has its origins in a course Mahajan, who has a PhD in physics, began teaching at MIT three years ago during the Independent Activities Period (IAP), the winter-break session in which students can take extra courses that emphasize hands-on learning. He says the book is intended for “students, practicing engineers, scientists, anyone who has to use mathematics to solve problems and get rough answers quickly.”

Given its practical focus, *Street-Fighting Mathematics* is not organized around traditional math topics, such as differential equations, but ways of thinking: reasoning by analogy, visualizing geometric problems, and more. Readers can then answer all manner of questions: Guessing the number of babies in the United States, calculating the bond angles in methane, or determining the drag that air exerts on a 747.

‘Math is not a spectator sport’

Mathematicians with an interest in the public understanding of science are impressed with Mahajan’s effort. “There’s a certain bravery in Sanjoy’s book,” says Steven Strogatz, a professor of mathematics at

Cornell, and author of an ongoing series of columns on math at The New York Times online. “He comes out defiantly and says, Of course I’m being imprecise — then you fill in the details. I wish more people in math would do that. Math is not a spectator sport. It’s an active enterprise. That’s what we all know, but we don’t all teach it that way.”

To make math an active enterprise for his students, Mahajan requires that they give him feedback about the course readings in advance of his lectures. Building on the work of Sacha Zyto, a PhD student in mechanical engineering, Mahajan has helped develop an online system in which students annotate the reading materials via PDF files. This method helps Mahajan evaluate the clarity of his presentations and see which ideas stymie students most often, while even allowing students to answer other students’ queries.

“You want the students to wrestle with the material, to make the knowledge their own,” says Mahajan.

Mahajan’s unconventional teaching practices stem from his focus, as a physicist, on finding quick, practical answers. Then again, perhaps rolling up one’s sleeves and hacking through problems is how everyone works. “There is a culture in pure mathematics that emphasizes rigor and careful proofs,” says Strogatz. “Yet all practicing mathematicians know we also use our intuitions, then we clean our answers up.” Strogatz is hopeful that Street-Fighting Mathematics can help form a pedagogical trend away from rote learning, and toward a more practical approach.

So let’s get back to the initial question (the numbers relate to the storage capacity of a data CD-ROM). The key to solving it, says Mahajan, is to recognize that the components of the first, messy-looking number can be broken into powers of 10. Then we can temporarily set aside these powers of 10 — Mahajan calls this “taking out the big part,” one of his tenets of problem-solving — while handling the smaller, simpler multiplication problem.

Okay: Picture the number as $(3.6 \times 10^3) \times (4.4 \times 10^4) \times (3.2 \times 10^1)$. To multiply powers of 10 in practice, we add them, here producing 108. Leave that aside momentarily and multiply 3.6 x 4.4 x 3.2. The answer is about 50, or 5.0×10^1. Combine that with 10^8, and we have our answer: Roughly 5.0×10^9, which is bigger than 3×10^9. Street-fighting math, and we barely got a scratch.

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