Chern numbers of algebraic varieties

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A problem at the interface of two mathematical areas, topology and algebraic geometry, that was formulated by Friedrich Hirzebruch, had resisted all attempts at a solution for more than 50 years. The problem concerns the relationship between different mathematical structures. Professor Dieter Kotschick, a mathematician at the Ludwig-Maximilians-Universität (LMU) in Munich, has now achieved a breakthrough. As reported in the online edition of the journal *Proceedings of the National Academy of Sciences* of the United States of America (*PNAS*), Kotschick has solved Hirzebruch's problem.

Topology studies flexible properties of geometric objects that are unchanged by continuous deformations. In algebraic geometry some of these objects are endowed with additional structure derived from an explicit description by polynomial equations. Hirzebruch's problem concerns the relation between flexible and rigid properties of geometric objects.

Viewed topologically, the surface of a ball is always a sphere, even when the ball is very deformed: precise geometric shapes are not important in topology. This is different in algebraic geometry, where objects like the sphere are described by polynomial equations. Professor Dieter Kotschick has recently achieved a breakthrough at the interface of topology and algebraic geometry.

"I was able to solve a problem that was formulated more than 50 years ago by the influential German mathematician Friedrich Hirzebruch", says Kotschick. "Hirzebruch's problem concerns the relation between

different mathematical structures. These are so-called algebraic varieties, which are the zero-sets of polynomials, and certain geometric objects called manifolds." Manifolds are smooth topological spaces that can be considered in arbitrary dimensions. The spherical surface of a ball is just a two-dimensional manifold.

In mathematical terminology Hirzebruch's problem was to determine which Chern numbers are topological invariants of complex-algebraic varieties. "I have proved that - except for the obvious ones - no Chern numbers are topologically invariant", says Kotschick. "Thus, these numbers do indeed depend on the algebraic structure of a variety, and are not determined by coarser, so-called topological properties. Put differently: The underlying manifold of an algebraic variety does not determine these invariants."

The solution to Hirzebruch's problem is announced in the current issue of *PNAS* Early Edition, the online version of *PNAS*.

Source: Ludwig-Maximilians-Universität München