Finding out what the Big Bang and ink jets have in common

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It often turns out there is more to commonplace everyday events than meets the eye. The folding of paper, or fall of water droplets from a tap, are two such events, both of which involve the creation of singularities requiring sophisticated mathematical techniques to describe, analyse and predict. On the positive side, there is much in common between many such singular events across the whole range of scales, from microscopic interactions to the very formation of the universe itself during the Big Bang.

In the past these seemingly unconnected events involving singularities have tended to be studied in isolation by different scientists with relatively little interaction or exchange of ideas between them.

Singularities occur at a point of cut off, or sudden change, within a physical system, as in formation of cracks, lightning strikes, creation of ink drops in printers, and the breaking of a cup when it drops. Improved understanding of the underlying mathematics would have many benefits, for example in making materials of all kinds that are more resistant to cracking or breaking. A recent workshop organised by the European Science Foundation (ESF) represented one of the first attempts to unify the field of singularities by bringing together experts in the different fields of application from astronomy to nanoscience, to develop common mathematical approaches.

"Singularities represent a subject that cuts across disciplines and specializations, such as experimental physics, theoretical physics, and rigorous mathematical proofs," noted the workshop's convenor Jens Eggers. "This workshop very much reflected this fact, as we had speakers from very different backgrounds."

The workshop confirmed that most if not all singular events in the universe, ranging from microscopic cracks to the Big Bang, share one important property known as self-similarity. This means that under magnification the event looks almost the same. For example a crack in a piece of plastic exhibits the same jagged structure when magnified say 100 times. This enables common mathematical approaches to be applied.

However it is also true that the "devil lies in the detail" when it comes to comparing different types of singularity. In other words different systems might have some common features such as self-similarity, but also unique aspects that require specialised study. One aim of the workshop therefore was to identify the common methods that could be applied as a foundation for more detailed specific study of a particular type of singularity.

This was reflected in the wide range of systems discussed. One such system, dealing with cracks in structures or rock formations, was presented by Jay Fineberg from the Hebrew University in Jerusalem. He talked about new experiments involving gels, allowing the structure of the crack to be determined in great detail down to very small microscopic dimensions, yielding some unexpected findings. "In particular, the structure of a crack is often more complicated than anticipated. Instead of one single crack path, the crack splits and has many small side branches, which appear to have complicated, if not fractal, structure," said Eggers. Fractal structure here means much the same as self-similarity, involving a geometrical pattern that looks unchanged under magnification or reduction.

Another example of everyday relevance concerned the singularities of crumpling in paper, presented by Tom Witten from the James Franck Institute in Chicago. A crumpled piece of paper comprises many ridges and tips, which defy easy analysis. As Eggers noted, there are many unanswered questions even in describing each individual cone-shaped tip. Yet understanding the underlying mathematics would not just help understand what happens when we crumple up a piece of paper to throw away, but also other physical systems involving ridges and tips, such as the folding of proteins during their manufacture in biological cells.

One question might be what the connection is between singularity theory, and catastrophe theory, which came to prominence in the 1970s, initially developed by French mathematician René Thom and then expanded by UK mathematician Erik Zeeman. In fact catastrophe theory is a sub-branch of singularity theory, dealing with events within physical space-time, such as collisions between wave fronts, as Eggers pointed out. "In that case, a problem that takes place in all of space can be reduced to a problem that takes place along certain lines (caustics), which can be classified according to catastrophe theory," said Eggers. However this simplification cannot be applied to all singularity problems.

Source: European Science Foundation