Glimpses of a new (mathematical) world

March 13 2008

A new mathematical object was revealed yesterday during a lecture at the American Institute of Mathematics (AIM). Two researchers from the University of Bristol exhibited the first example of a third degree transcendental L-function. These L-functions encode deep underlying connections between many different areas of mathematics.

The news caused excitement at the AIM workshop attended by 25 of the world's leading analytic number theorists. The work is a joint project between Ce Bian and his adviser, Andrew Booker. Booker commented that, "This work was made possible by a combination of theoretical advances and the power of modern computers." During his lecture, Bian reported that it took approximately 10,000 hours of computer time to produce his initial results.

"This breakthrough opens a door to the study of higher degree L-functions," said Dennis Hejhal, Professor of Mathematics at the University of Minnesota and Uppsala University.

"It's a big advance' added Harold Stark of the University of California, San Diego, who, 30 years ago was the first to accurately calculate second degree transcendental L-functions.

"I thought we were years away from doing this. The geometry of what you have to do and the scale of the computation are orders of magnitude harder."

There are two types of L-functions: algebraic and transcendental, and

these are classified according to their degree. The Riemann zeta-function is the grand-daddy of all L-functions. It holds the secret to how the prime numbers are distributed, and is a first-degree algebraic L-function.

The Riemann Hypothesis, announced in 1859 and today the most important of all unsolved math problems, is an example of something that should be true for EVERY L-function. Michael Rubinstein from the University of Waterloo, a participant at the workshop, quickly tested and confirmed the Riemann Hypothesis for the first few zeroes of this newly minted L-function.

Rubinstein, along with William Stein of the University of Washington, will direct a new initiative to chart all L-functions; this project has been recommended for funding by the National Science Foundation. "The techniques developed by Bian and Booker open up whole new possibilities for experimenting with these powerful and mysterious functions and are a key step towards making our group project a success." Rubinstein added.

"It's a big step toward our understanding the 'world of L', which is where most of the secrets of number theory are kept." said Brian Conrey, Director of AIM.

Dorian Goldfeld, Professor of Mathematics at Columbia University summarized the excitement, saying "This discovery is analogous to finding planets in remote solar systems. We know they are out there, but the problem is to detect them and determine what they look like. It gives us a glimpse of new worlds."

Source: American Institute of Mathematics