# A crystal that nature may have missed

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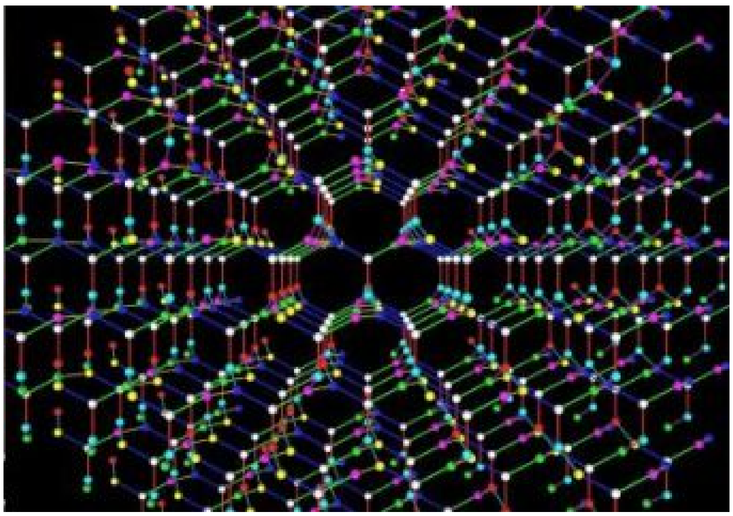

K_4 crystal. Created by Hisashi Naito. Credit: Hisashi Naito

For centuries, human beings have been entranced by the captivating glimmer of the diamond. What accounts for the stunning beauty of this most precious gem? As mathematician Toshikazu Sunada explains in an article appearing today in the *Notices of the American Mathematical Society*, some secrets of the diamond's beauty can be uncovered by a mathematical analysis of its microscopic crystal structure.

It turns out that this structure has some very special, and especially symmetric, properties. In fact, as Sunada discovered, out of an infinite universe of mathematical crystals, only one other shares these properties with the diamond, a crystal that he calls the "K_4 crystal". It is not known whether the K_4 crystal exists in nature or could be synthesized.

One can create an idealized mathematical model of a crystal by focusing on its main features, namely, the atoms and the bonds between them. The atoms are represented by points, which we will call "vertices", and the bonds are represented as lines, which we will call "edges". This kind of network of vertices and edges is called a "graph". A crystal is built up by starting with a building-block graph and joining together copies of itself in a periodic fashion.

Thus there are two patterns operating in a crystal: The pattern of edges connecting vertices in the building-block graphs (that is, the pattern of bonding relations between the atoms), and the periodic pattern joining the copies of the graphs. One can create infinitely many mathematical crystals this way, by varying the graphs and by varying the way they are joined periodically.

The diamond crystal has two key properties that distinguish it from other crystals. The first, called "maximal symmetry", concerns the symmetry of the arrangement of the building-block graphs. Some arrangements have more symmetry than others, and if one starts with any given arrangement, one can deform it, while maintaining periodicity and the bonding relations between the atoms, to make it more symmetrical. For the diamond crystal, it turns out that no deformation of the periodic arrangement can make it any more symmetrical than it is. As Sunada puts it, the diamond crystal has maximal symmetry.

Any crystal can be deformed into a crystal with maximal symmetry, so that property alone does not distinguish the diamond crystal. But the diamond crystal has a second special property, called "the strong isotropic property". This property resembles the rotational symmetry that characterizes the circle and the sphere: No matter how you rotate a circle or a sphere, it always looks the same. The diamond crystal has a similar property, in that the crystal looks the same when viewed from the direction of any edge. Rotate the diamond crystal from the direction of one edge to the direction of a different edge, and it will look the same.

It turns out that, out of all the crystals that are possible to construct mathematically, just one shares with the diamond these two properties. Sunada calls this the K_4 crystal, because it is made out of a graph called K_4, which consists of 4 points, in which any two vertices are connected by an edge.

"The K_4 crystal looks no less beautiful than the diamond crystal," Sunada writes. "Its artistic structure has intrigued me for some time." He notes that, although the K_4 crystal presently exists only as a mathematical object, it is tempting to wonder whether it might occur in nature or could be synthesized. This is not so far-fetched as it may sound: The Fullerene, which has the structure of a soccer ball (technically called a truncated icosahedron), was identified as a mathematical object before it was found, in 1990, to occur in nature as the C_60 molecule.

Sunada's article, "Crystals That Nature Might Miss Creating", is appearing in the February 2008 issue of the *AMS Notices* and is being posted online today at www.ams.org/notices .

Source: American Mathematical Society

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