Relativity Derived Without Calculus -- Possibly Centuries Ago

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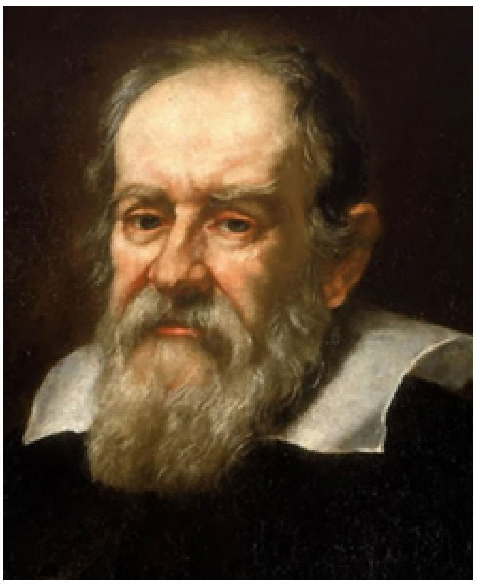

Galileo discovered “Galilean relativity.” He had the tools to allow him—in theory—to discover Einsteinian relativity, as well. (Original portrait of Galileo painted by Justus Sustermans in 1636.)

After Einstein developed his theories of special and general relativity, in 1905 and 1916, respectively, the world of physics changed dramatically. The theories, with their groundbreaking ideas on space and time, helped lead 20th century scientists to unlock the secrets of the atom and unleash the power of nuclear energy.

Einsteinian relativity seemed to be a modern breakthrough: he had derived his theories from ideas and mathematics that were new at the time. The Lorentz transformations had just been discovered in 1895, and he derived a new velocity addition law using calculus (both of these concepts describe how observers in different reference frames perceive each other). Further, Einstein based his theories on the assumption that the speed of light, c, is constant, and used *gedanken* ("thought") experiments involving light rays to reach his conclusions.

Now Joel Gannett, a Senior Scientist in the Applied Research Area of Telcordia Technologies in Red Bank, New Jersey, has found that Einstein didn't have to do the work the hard way. A researcher in optical networking technologies, Gannett has shown that the Lorentz transformations and velocity addition law can be derived without assuming the constancy of the speed of light, without thought experiments, and without calculus. In this case, Einsteinian relativity could have been discovered several centuries before Einstein.

"Einsteinian Relativity is difficult to wrap your mind around," Gannett told *PhysOrg.com*. "It does not help that Einstein's seminal 1905 paper, and many discussions of the topic since, start off with the wildly counterintuitive assumption that the speed of light is constant in all inertial frames.

"My work shows that the essential strangeness of Einsteinian Relativity falls out of simple, intuitive assumptions using simple math. A pre-calculus high school student could have derived Einsteinian Relativity. Admittedly, some of the math in my paper might seem beyond the high school level, but that was because I was proving continuity from a boundedness assumption. One could bypass this math by simply assuming continuity, a logical step that would probably feel comfortable to most any high schooler or 17th century scientist."

Gannett is not the first person to suggest that a simpler path to modern relativity might exist. In 2003, Palash Pal of the Saha Institute of Nuclear Physics in Calcutta, India showed that the Lorentz transformations could be derived without assuming the constancy of c and without thought experiments; in fact, scientists had noted this possibility as far back as 1910.

To reach his derivation, Pal invoked the ideas that spacetime is homogeneous and isotropic. Pal titled his paper “Nothing but relativity”; after reading it, Gannett has called his paper “Nothing but relativity, redux,” which is published in a recent issue of the *European Journal of Physics*. However, Gannett explains that his derivation actually bypasses the principle of relativity altogether—instead, he assumes the simpler idea of reciprocity.

“The current paper might have been titled ‘Nothing but relativity, and not that either,’ or perhaps ‘Nothing but reciprocity,” he writes, emphasizing the point.

“One of the issues I raise in my paper is, why make a heavyweight assumption such as relativity when in fact all you need for the derivation is reciprocity?” he explains. “I don't need the fact that the laws of physics are the same for you on the speeding train and me on the platform (i.e., relativity). All I need is reciprocity.”

Gannett uses the common analogy of the train to explain reciprocity: “Suppose you are on a train and I am on the platform waving goodbye. Suppose I measure your speed relative to me as 30 mph. Looking back at me, you would judge that I am moving away from you at 30 mph as well. If we both had police radar guns to measure our relative speeds with great accuracy, we would both come up with the same number (say, 29.6 mph). That's reciprocity. In the presence of isotropy, which is one of my other key assumptions, relativity implies reciprocity.”

Instead of using calculus to derive the spacetime transformations, Gannett uses two basic concepts from mathematical topology: density and continuity. Using these concepts, he demonstrates how the spacetime coordinates of one reference frame (e.g. the train) can be mapped to the spacetime coordinates of a second reference frame (e.g. the platform), accounting for the distorted lengths and times that occur at high speeds.

The density concept means that any irrational number (such as the number π) can be approximated to arbitrary closeness by a rational number (those with a finite number of digits or digits that repeat). The second concept, continuity, means that a function maps “close-by” points to “close-by” points. From these concepts he derives a linear homogeneous function, or a matrix, to connect the coordinates of the two reference frames.

Gannett explains that proving that spacetime is linear is a vital point to make before reaching Pal’s derivation. He also notes that Einstein had glossed over this important point in his paper on special relativity.

“Einstein merely stated that homogeneity (i.e., the uniformity of space and time) clearly implied linearity,” he says. “With the level of mathematics I was applying in my paper, I could fairly easily get to the point where one could assert linearity in a mythical universe where coordinates exist only as rational numbers, and we consider only rational scalings. But since the days of Pythagoras this would not be considered adequate. Because rational numbers can approximate irrational numbers to any desired degree of accuracy, continuity provides the final logical link that lets you assert linearity of the spacetime mapping.”

Finally, invoking the ideas of the cosmological principle that the universe is isotropic and reciprocal, Gannett demonstrates that four basic properties of the mapping functions directly follow. From that point, Pal’s equations finish deriving the Lorentz transformation and velocity addition law using only algebra and basic physical considerations.

What if Galileo, back in the 17th century, had derived the principles of Einstein's relativity? Would the history of science progress been different? Gannett doesn't think so.

"If Galileo had derived it, Occam's razor ['the law of simplicity'] would have impelled him to discard Einsteinian relativity as a needlessly complex mathematical curiosity that was not required to resolve any outstanding issues known to 17th century science," he predicts.

In Einstein's time, things were different. Newton's classical mechanics clashed with Maxwell's equations of electromagnetism, which date from 1861. With the ideas in special relativity, which showed that electrodynamics obeyed relativity, Einstein replaced the old spacetime model from Newtonian mechanics and solved a major challenge of the early 20th century.

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