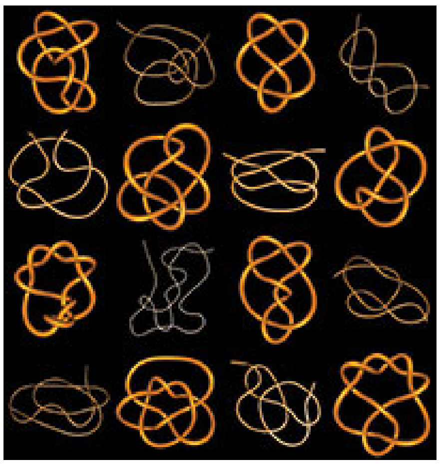
Physicists Tackle Knotty Puzzle

October 1 2007



Digital photos of knots with computer-generated drawings based on mathematical calculations. Image credit: Dorian Raymer, UCSD

Electrical cables, garden hoses and strands of holiday lights seem to get themselves hopelessly tangled with no help at all. Now research initiated by an undergraduate student at the University of California, San Diego has resulted in the first model of how knots form.

The study, published this week in the journal *Proceedings of the National Academy of Sciences*, investigated the likelihood of knot formation and the types of knots formed in a tumbled string. The researchers say they were interested in the problem because it has many applications, including to the biophysics research questions their group usually studies.

“Knot formation is important in many fields,” said Douglas Smith, an assistant professor of physics who was the senior author on the paper. “For example, knots often form in DNA, which is a long string-like molecule. Cells have enzymes that undo the knots by cutting the DNA strands so that they can pass through each other. Certain anti-cancer drugs stop tumor cells from dividing by blocking the unknotting of DNA.”

Dorian Raymer, a research assistant working with Smith, initiated the study because he was interested in knot theory—the branch of mathematics that uses formulae to distinguish unique knots. Raymer was an undergraduate major in physics when he did the work. Smith said his own interest was piqued when he discovered that no one really knew how knots formed.

“Very little experimental work had been done to apply knot theory to the analysis and classification of real, physical knots,” said Smith. “For mathematicians, the problem is very abstract. They imagine the types of knots that can form and then classify them. In our experiments, we produced thousands of different knots, used mathematical knot theory to analyze them, and then developed a simple physics model to explain our findings.”

The experimental set up consisted of a plastic box that was spun by a computer-controlled motor. A piece of string was dropped into the box and tumbled around like clothes in a dryer. Knots formed very quickly, within 10 seconds. The researchers repeated the experiment more than 3,000 times varying the length and stiffness of string, box size and speed of rotation. They classified the resulting knots.

“It is virtually impossible to distinguish different knots just by looking at them,” said Raymer. “So I developed a computer program to do it. The computer program counts each crossing of the string. It notes whether the crossing is under or over, and whether the string follows a path to the left or to the right. The result is a bunch of numbers that can be translated into a mathematical fingerprint for a knot.

“We used the Jones polynomial—a famous math formula developed by Vaughn Jones, a mathematics professor at U.C. Berkeley—because it automatically simplifies mirror images and other knots that are identical, but look different.”

Rather than getting just a few types of knots, Smith and Raymer got all the types that mathematicians had enumerated, at least up to a certain complexity level. The longer the string, the greater was the probability of getting complex knots.

Based on these observations, the researchers proposed a simplified model for knot formation. The string forms concentric coils, like a looped garden hose, due to its stiffness and the confinement of the box. The free end of the string weaves through the coils, with a 50 percent probability of going under or over any coil. A computer simulation based on this model produced a similar pattern of simple and complex knots as observed in their experiments.

Smith and Raymer said that the model can also explain why confining a stiff string in a smaller box decreases the probability of knot formation. Increased confinement reduces the tumbling motion that facilitates the weaving of the string end through the coils. The paper cites other researchers who have proposed a similar effect to explain why knotting of the umbilical cord of fetuses is relatively rare, occurring only about one percent of the time. Confinement to the amniotic sac may restrict the probability of knotting.

Smith said that their results do not point to any magic solution to prevent knots from forming, but the project did inspire some advice for young people interested in science.

“Even today, there are still interesting scientific problems that can be studied in your garage with inexpensive, off-the-shelf materials like the ones we used in our experiments,” he said. “The most important thing is to be curious and ask good questions.”

Source: UCSD

Citation: Physicists Tackle Knotty Puzzle (2007, October 1) retrieved 18 April 2024 from https://phys.org/news/2007-10-physicists-tackle-knotty-puzzle.html