Works of mathematical power, beauty yield Clay Research Prize

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In rational billiards, mathematicians examine the behavior of billiard balls on unusually shaped frictionless tables. Alex Eskin, Professor in Mathematics and the College, has received the Clay Research Prize for his research on rational billiards and other work. Moon Duchin, a former graduate student of Eskin&acutes, ran across this L-shaped table at a sports bar near Gansbai, South Africa, in 2001. (Photo courtesy of Moon Duchin)

An institute that promotes the "beauty, power and universality" of mathematical thought has awarded the Clay Research Prize to Alex Eskin, Professor in Mathematics at the University of Chicago.

"Eskin has exhibited many times the ability to pound through incredible walls that look impassable," said Benson Farb, Professor in Mathematics at Chicago. “He is simply fearless.”

Eskin is one of five recipients of the 2007 Clay Research Prize, presented by the Clay Mathematics Institute of Cambridge, Mass. In describing his work, Eskin’s colleagues refer repeatedly to the aesthetics of his mathematics. He has produced the numerical equivalent of a stirring poem or a melodic symphony, said Kevin Corlette, Professor in Mathematics at Chicago.

The Clay Institute cited Eskin for work in two completely different areas, rational billiards and geometric group theory. In rational billiards, Eskin tackled a long-standing problem related to randomness. “Consider a billiard ball on a frictionless polygonal table,” Eskin said. “If the ball is set in motion, it will travel forever, making perfectly elastic collisions with the walls.”

On such a table shaped like a square or an equilateral triangle, mathematicians know that a ball either repeats the same path forever, or it travels randomly over the entire polygon. Eskin’s research addresses other types of polygons, where the ball’s behavior differs.

The corner angles in these polygons may measure 12.5 degrees, for example, or 93 and two-thirds degrees. How would that affect the path of the ball? “There are deep connections between this problem and many other areas of mathematics,” Eskin said.

Eskin described geometric group theory as “very roughly the study of objects when viewed from far away.” His fascination with the topic reflects his fondness for “understanding the most basic objects in more and more profound ways,” said Shmuel Weinberger, Professor in Mathematics at Chicago.

A constellation of current and former Chicago mathematicians figure into Eskin's equation for success in this field. The Clay Institute noted his "crucial contributions" to proving a theorem jointly with David Fisher and Kevin Whyte.

Fisher (Ph.D.,'99), who was a Ph.D. student of University of Chicago President Robert Zimmer when Zimmer was the Max Mason Distinguished Service Professor in Mathematics, is an assistant professor in mathematics at Indiana University. Whyte (Ph.D. '98), who was a student of Weinberger's, is an associate professor in mathematics at the University of Illinois at Chicago. Together the three mathematicians solved several conjectures in geometric group theory proposed by Farb and Lee Mosher of Rutgers University, Newark.

A key piece to solving some of these conjectures emerged from the thesis of Tullia Dymarz, a student of Farb's who will receive her Ph.D. in June. And Irine Peng, a Ph.D. student in Mathematics who works with Eskin, extended some of the Eskin-Fisher-Whyte work to a broader class of examples, Farb said.

Eskin comes from a long line of mathematicians who have fruitfully studied geometry through the framework of Lie Groups. One class of problems in the theory of Lie Groups goes by the name of "semisimple." In the 1990s, research on semisimple Lie groups has produced "an explosion of progress," said Farb.

Not so in another class of vexing Lie group problems, rather ironically known as "solvable groups." Despite repeated attempts at solving these problems, mathematicians have enjoyed little progress in recent decades.

No known techniques proved capable of solving Farb and Mosher's conjectures regarding a broad class of solvable groups. Mathematicians thus focused their attention on one of the simplest solvable Lie groups, the three-dimensional Sol group.

Weirdly behaving light rays come into play here. In the Euclidean geometry of everyday life, a straight line marks the shortest distance between two points. But in the seemingly impossible geometry of Sol, the shortest distance between two points sometimes follows a straight line, sometimes a spiraling path.

Furthermore, some pairs of points in Sol even have more than one shortest path between them. This is because Sol exhibits a subtle mix of both the positive curvature of a sphere and the negative curvature of a saddle. "The geometry of this space is bizarre and fascinating; even after years of study it can continue to surprise," Farb said.

Farb thought hard about the Sol problem, "but at the end of each new approach, I ended up right back at square one," he said. He gave up in frustration and moved on to other problems.

Eskin also worked the problem. He would drop by Farb's office every six months with a new idea, to no avail. "But Eskin is relentless," Farb said.

"In 2005, Eskin walked into my office and started explaining to me a number of new, really beautiful ideas about how to solve the problem," Farb recalled. "It became clear that this might be the key."

Eventually, Eskin, Fisher and Whyte combined methods from several areas of mathematics in ingenious ways to solve the Sol problem, most of the Farb-Mosher conjectures, and then some. "The theorem on Sol shows that from some points of view, solvable groups behave similarly to the rest, but for completely different reasons," Weinberger said. "It really opens up whole new vistas."

Source: University of Chicago