Composer reveals musical chords' hidden geometry

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Composers often speak of fitting chords and melodies together, as though sounds were physical objects with geometric shape -- and now a Princeton University musician has shown that advanced geometry actually does offer a tool for understanding musical structure.

In an attempt to answer age-old questions about how basic musical elements work together, Dmitri Tymoczko has journeyed far into the land of topology and non-Euclidean geometry, and has returned with a new -- and comparatively simple -- way of understanding how music is constructed. His findings have resulted in the first paper on music theory that the journal *Science* has printed in its 127-year history, and may provide an additional theoretical tool for composers searching for that elusive next chord.

"I'm not trying to tell people what style of music sounds good, or which composers to prefer," said Tymoczko (pronounced tim-OSS-ko), a composer and music theorist who is an assistant professor of music at Princeton. "What I hope to do is provide a new way to represent the space of musical possibilities. If you like a particular chord, or group of notes, then I can show you how to find other, similar chords and link them together to form attractive melodies. These two principles -- using attractive chords, and connecting their notes to form melodies -- have been central to Western musical thought for almost a thousand years."

Tymoczko's findings appear as a report in Science's July 7 issue.

Making graphical representations of musical ideas is not itself a new idea. Even most nonmusicians are familiar with the five-line musical staff, on which the notes that appear physically higher represent sounds that have higher pitch. Other common representations include the circle of fifths, which illustrates the relationships between the 12 notes in the chromatic scale as though they were the 12 hours on a clock's face.

"Tools like these have helped people understand music with both their ears and their eyes for generations," Tymoczko said. "But music has expanded a great deal in the past hundred years. We are interested in a much broader range of harmonies and melodies than previous composers were. With all these new musical developments, I thought it would be useful to search for a framework that could help us understand music regardless of style."

Traditional music theory required that harmonically acceptable chords be constructed from notes separated by a couple of scale steps -- such as the major chord, whose three notes comprise the first, third and fifth elements in the major scale, forming a familiar harmony that most audiences find easy to enjoy. Many 20th-century composers abandoned this requirement, however. Modern chords are often constructed of notes that sit right next to one another on the keyboard, forming "clusters" -- dissonant by traditional standards -- that to this day often challenge listeners' ears.

"Western music theory has developed impressive tools for thinking about traditional harmonies, but it doesn’t have the same sophisticated tools for thinking about these newer chords," Tymoczko said. "This led me to want to develop a general geometrical model in which every conceivable chord is represented by a point in space. That way, if you hear any sequence of chords, no matter how unfamiliar, you can still represent it as a series of points in the space. To understand the melodic relationship between these chords, you connect the points with lines that represent how you have to change their notes to get from one chord to the next."

One of Tymoczko's musical spaces resembles a triangular prism, in which points representing traditionally familiar harmonies such as major chords gather near the center of the triangle, forming neat geometric shapes with other common chords that relate to them closely. Dissonant, cluster-type harmonies can be found out near the edges, close to their own harmonic kin. Tymoczko said that composers have traditionally valued a kind of harmonic consistency that does not require that the listener jump far from one region of the space to another too quickly.

“This idea that you should stay in one part of space,” he said, “is an important ingredient of our notion of musical coherence.”

To bring these ideas to life, Tymoczko has created a short movie that illustrates the chord movement in a piece of music by 19th-century composer Frederick Chopin. His E minor piano prelude (Opus 28, No. 4) has charmed listeners since the 1830s, but its harmonies have not been well explained.

"This prelude is mysterious," Tymoczko said. "While it uses traditional harmonies, they are connected with nonstandard chord progressions that people have had trouble describing. However, when you plot the chord movement in geometric space, you can see Chopin is moving along very short lines, staying primarily within one region."

Tymoczko said that the geometric approach could assist with our still-murky understanding of music ranging from the mid-1800s through the contemporary period, including the cluster-based compositions of Georgi Ligeti, whose work formed a dramatic part of the soundtrack to the film "2001: A Space Odyssey."

"What all this implies is that you can begin with any sort of harmony your ear enjoys, whether it's a familiar chord from a 300-year-old hymn or the most avant-garde cluster you can imagine," he said. "But once you have decided where to start from and what region of space your harmony inhabits, very general principles of musical coherence suggest that you stay close to that region of space."

Tymoczko, whose compositional influences include classical music, rock and jazz, said he does not expect people will start writing music by "connecting the dots" as a result of his research. But he hopes it will at least provide a new tool for understanding the relationships behind music.

"Put simply, I'm a composer and I like to write and play music that sounds good," he said. "But what does it mean to 'sound good'? That's a question that the musical community has grappled with for centuries. Our understanding of the Chopin piece, for example, had previously been very local -- as if we were walking in a heavy fog and could only see a few steps in front of our feet at any one time. We now have a map of the whole terrain on which we can walk, and can replace our earlier, local perspective with a much more general one."

Commenting on the significance of the work, Yale's Richard Cohn said that Tymoczko has made a useful contribution to a fundamental problem in music theory.

"Dmitri's solution is exhaustive, original, and expressed clearly enough to be meaningful even to those musicians and scholars who do not have Dmitri's mathematical
abilities," said Cohn, who is the Batell Professor of the Theory of Music at Yale. "His work leads to a deeper understanding of why composers in the European tradition favor certain types of scales and chords, and it suggests that melody and harmony are more fundamentally intertwined

than has been previously thought. His achievement will become central to future work in the modelling of musical systems."

Source: Princeton University, by Chad Boutin