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New twist in classical mechanics finds way around 225-year-old paradox

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In the rarefied sphere of classical mechanics, more can sometimes be elegantly less.
In a paper that will be published March 1 in the proceedings of the Royal Society, two engineers at the Viterbi School of Engineering offer a new and potentially much more flexible method of mathematically describing mechanical systems.

The method also resolves a more than 200-year-old mathematical paradox, according to Professor Firdaus Udwadia, who co-wrote the paper with his former PhD student Phailaung Phomosiri.

The paradox and the problem both come from classic work of the French physicist Joseph Louis Lagrange (1736-1813) who in 1788 described a new mathematical way to represent the movement of systems of connected mechanical parts ("constrained mechanical systems"), one simpler than that originally devised by Newton a century before.

Udwadia gives a simple example of such a system: a double pendulum, a weight hanging on a link, with a second weight attached to tit by another link.

The angle of the first weight's link from the vertical, the length of the that link; the angle and length of the second link, plus the masses of the weights all combine into a mathematical matrix, which when solved using classic Lagrange/Gauss methods, describe the system. But the calculations can be formidably complex.

Udwadia's new method involves omitting the linkage that makes one system of two, representing the parts of the system with a set of coordinates representing first, their position in space, omitting the linkage.

Udwadia says this description can then be considerably simplified if a single system is then decomposed into two or more separate ones. "You can ignore the link connecting weight 1 and weight 2, and just track weight 2 as a mass moving through space."

"You need more coordinates to describe the system as two separate ones instead of one," he continues. But more is less: "the calculations for the two separate systems themselves become much simpler."

Udwadia said that the difficulty with this method comes in mathematically reintroducing the missing link or links. The information necessary to meld the multiple systems back together into one is present, but reconstructing it in many cases leads to a mathematical dead end called a "singular mass matrix."

Such matrices have been known since the time of Lagrange: classically, you get one if you have an element in your machine that you say has zero mass. "Theoretically, it shouldn't affect the system at all," says Udwadia, "but the mathematical effect is that the Lagrangian framework goes into a tail spin, and cannot be used."

Lagrangian matrices are used in quantum mechanics as well as classical mechanics, and in 1964, quantum physicist Paul Dirac made a breakthrough. While studying constrained motion in quantum systems he discovered that in certain cases - in systems called Hamiltonians - he found a way to find correct equations of motion even though the matrices were singular.

Udwadia and Phomosiri also found a way around, though they obtained their equations of motion by a method completely different than Dirac's. They state in their paper that "the general, explicit equation of motion obtained in this paper that is applicable to systems with singular mass matrices with general, holonomic and nonholonomic constraints that may or may not be ideal, appears to be first of a kind in classical mechanics."

The authors add that "these equations, permit one to decompose complex multi-body systems into subsystems .. and then recombine these subsystem equations to obtain the equations of motion of the composite system in a straightforward and simple manner." Other areas of application may appear.

"Where other researchers will take this fundamental equation, is difficult to say," says Udwadia.

Udwadia is a professor in the USC Viterbi School departments of Civil and Environmental Engineering and Aerospace and Mechanical as well as holding appointments in Mathematics in the College, and in Information and Systems Management in the USC Marshall School of Business.

He has been pursuing work on Langrangian mechanics for more than a decade. In 1992, he and collaborator Robert Kalaba introduced an extension of Lagrange's work to non-holonomic systems. In 2001, Udwadia and Kalaba introduced a new way of dealing with problems of unconstrained motion.

Source: University of Southern California

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