The Math Of Deadly Waves

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When Walter Craig saw the images of the devastating 2004 Boxing Day Indian Ocean tsunami he felt compelled to act. So he grabbed a pencil and envelope and started calculating.

A little more than a year later, the mathematical analyst says that mathematics has a role to play in washing away misconceptions and myths about these deadly waves – and potentially saving lives.

"Predicting earthquakes is a grand challenge problem that's presently beyond us. But predicting a tsunami's potential based on these earthquakes is a doable problem and I think mathematicians can play an important role in this," says Dr. Craig, the Canada Research Chair for Mathematical Analysis and its Applications, at McMaster University in Hamilton, Canada.

"Mathematics is particularly well suited to defining the possibilities and limitations for a tsunami early warning system," says Dr. Craig. It's a conviction that's prompted him to co-organize the symposium on Tsunamis: Their Hydrodynamics and Impact on People at the annual meeting of the American Association for the Advancement of Science (AAAS) in St. Louis, on Sunday, February, 19.

Dr. Craig studies the mathematical theory of wave equations that are derived from physics. In collaboration with colleagues he has applied these theories to scientific problems large and small, from the quantum mechanical oscillations of electrons to the cosmic waves that rippled through the newborn universe. But rarely, he says, does the mathematics of wave propagation meet a subject so full of immediate human importance as with understanding rogue waves.

Mathematics, he says, has a key role to play in dispelling mistaken assumptions about these waves. One such popular belief is that a tsunami's first wave surge is always the biggest.

"It's not necessarily the biggest crest in front," he cautions. "For example, in Sri Lanka the biggest crest was the third or fourth." In one case, he says, a vacationing British geologist at one Sri Lankan resort noted the initial modest, non-destructive surge and warned staff and tourists to clear the beach before the arrival of the larger, deadly surges.

Dr. Craig says that mathematical modelling of the Indian Ocean tsunami showed it to be close to what he calls a "classical wave packet" – the wave behaved in a manner very close to that predicted by mathematical theory. It followed the pattern of a group of waves travelling together as well as evolving in form as they crossed the ocean basin.

Because of differences in depth, the evolution of a tsunami is different in different ocean basins. For example, the Boxing Day tsunami travelled twice as fast in the deeper Indian Ocean than in the Andaman Basin. Tsunami waves are distinguished from ordinary wind-generated ocean waves by their great length between peaks, often exceeding 200 kilometres in the deep ocean, and by the long amount of time between these peaks, ranging from 15 minutes to an hour.

It's the length and width of tsunamis, rather than their at-sea height that reveals their massive power. The Indian Ocean tsunami had a crest length of about 1,200 kilometres. The surges that inundated the Sri Lankan coast were parts of waves that were a stunning 100 kilometres from crest to trough, but in mid-ocean were less than one metre in amplitude.

"It's amazing to think about this. Even if the wave is only a metre high at mid-sea, this is a huge amount of water and it gives a sense of how much energy it's carrying," says Dr. Craig.

Another widely held belief about tsunamis that gets washed away with mathematical modelling is that the surge is always preceded by the tide going abnormally far out.

"This only happens about half the time," explains Dr. Craig. "It depends on the wavelength and whether it's the trough or crest of the wave that reaches shore first. In half the cases it's the surge that arrives first."

Dr. Craig acknowledges that for the most part geologists and tsunami experts have a strong practical understanding of how these giant waves behave. But, he says, given the paucity of real-world data on tsunamis, there are still many outstanding questions.

"To a first order of approximation the current modelling of a tsunami's evolution in mid ocean is very good," says Dr. Craig. "Nonetheless, there is much less known about the generation of tsunami waves, and about the amplification effects as they impact on coastal areas. These are not easy mathematical problems. Experimentally they're not seen very often, so it's still a question as to whether we're using the right equations to study them."

He's presently begun work with McMaster University mathematics colleagues Drs. Bartosz Protas and Nicholas Kevlahan to apply mathematical tools from meteorological forecasting to understand the generation of large tsunamis from major earthquakes. For example, some earthquakes generate large waves, while others of the same magnitude produce little or no wave response. Their approach will use hindcasting techniques – looking back over previous patterns to understand how we arrived at present conditions – to develop predictive computational models for tsunami sources.

While better advanced warning systems can help in many cases, Dr. Craig says his immersion in tsunami science has shown him that a tsunami's speed and power sometimes can defy an early warning system. With a wave traveling at 700 kilometres an hour, his advice is, "If you feel an earthquake, go to higher ground."