New mathematics-based sculpture unveils fourth dimension

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Artistic works traditionally carry significance beyond their physical beauty, but a new sculpture in the McAllister Building headquarters of the Penn State Department of Mathematics may carry that tradition to its limits.

Image: The sculpture, designed by Adrian Ocneanu, professor of mathematics at Penn State, presents a three-dimensional 'shadow' of a four-dimensional solid object.

The stainless-steel work, a striking object of visual art, also is a mental portal to the fourth dimension, a teaching tool, a memorial to a graduate of the math department, and a reminder of the terrorist attacks of Sept. 11, 2001. The sculpture itself measures about six feet in every direction and is mounted on a granite base about three feet high in order to bring its center approximately to eye level.

The sculpture, designed by Adrian Ocneanu, professor of mathematics at Penn State, presents a three-dimensional "shadow" of a four-dimensional solid object. Ocneanu's research involves mathematical models for quantum field theory based on symmetry. One aspect of his work is modeling regular solids, both mathematically and physically.

In the three-dimensional world, there are five regular solids -- tetrahedron, cube, octahedron, dodecahedron, and icosahedron -- whose faces are composed of triangles, squares or pentagons. In four dimensions, there are six regular solids, which can be built based on the symmetries of the three-dimensional solids. Unfortunately, humans cannot process information in four dimensions directly because we don't see the universe that way. Although mathematicians can work with a fourth dimension abstractly by adding a fourth coordinate to the three that we use to describe a point in space, a fourth spatial dimension is difficult to visualize. For that, models are needed.

"Four-dimensional models are useful for thinking about and finding new relationships and phenomena," said Ocneanu. "The process is actually quite simple -- think in one dimension less." To explain this concept, he points to a map. While the Earth is a three-dimensional object, its surface can be represented on a flat two-dimensional map.

Ocneanu's sculpture similarly maps the four-dimensional solid into a space perceptible to the human observer. His process, radial stereography, presents a new way of making this projection. He explained the process by analogy to mapping a globe of the Earth onto a flat surface.

"We place a light bulb at the north pole of the Earth and we project onto a sheet of paper placed underneath it," he said. "The southern hemisphere, away from the north pole, will remain quite small, while the northern hemisphere, near the projection pole, will become very big and north pole itself will be sent toward infinity."

The technique can be used to make a two-dimensional projection of a cube by first mapping the cube radially onto the surface of a globe. Ocneanu explained, "The edges of our cube become circles on the map, just like straight highways are slightly curved on maps of the Earth. Its angles, however, remain true in this projection, so the map retains the key aspects of the symmetry of the original cube, unlike a photograph of a cube."

When the same technique is applied to project a four-dimensional solid into three dimensions, the inner part of the projection -- equivalent to the south pole on the map -- has smaller, undistorted faces, while the outer part extends toward infinity. Linear edges of the solid become circles in the projection.

However, the projection is conformal, which means that the angles between faces and the way that the faces meet at corners are uniform throughout the projection. The retention of these key characteristics makes the sculpture a powerful teaching tool in addition to a powerful esthetic object.

"When I saw the actual sculpture, I had quite a shock," said Ocneanu. "I never imagined the play of light on the surfaces. There are subtle optical effects that you can feel but can't quite put your finger on." The sculpture has significance in several areas of mathematics related to the study of symmetry, and it can represent structures that are fundamental to many branches of mathematics and physics.

"The sculpture is a new way to represent a classical mathematical object," said Nigel Higson, head of the Penn State Department of Mathematics. "For professionals the sculpture is very rich in meaning, but it also has an aesthetic appeal that anyone can appreciate. In addition, it helps to start conversations about abstract mathematical concepts -- something that is generally hard to do with anyone other than another expert."

The subject of the projection is a regular 4-dimensional solid of intermediate complexity, which Ocneanu calls an "octacube." It has 24 vertices, 96 edges and 96 triangular faces, which enclose 24 three-dimensional "rooms." Windows cut in faces allow the viewer to see within the structure, the same way that a window in a cubic room opens to the inside of the cube. Physically, the sculpture is a giant puzzle of 96 triangular pieces cut from stainless steel and bent into spherical shape.

Ocneanu attributes the success of the project to the machinists and welders of Penn State's Engineering Services Shop, managed by Jerry Anderson. "It turned out way better than I could have imagined," Ocneanu said. "It's very hard to make 12 steel sheets meet perfectly -- and conformally -- at each of the 23 vertices, with no trace of welding left. The people who built it are really world-class experts and perfectionists -- artists in steel."

The sculpture was sponsored by Jill Grashof Anderson, a 1965 graduate of the mathematics department, who provided funds for its development and construction. It is dedicated to the memory of her husband, Kermit Anderson -- also a 1965 mathematics graduate -- who was killed in the World Trade Center terrorist attack on Sept. 11, 2001. She also has sponsored a scholarship in his memory.

"I hope that the sculpture will encourage students, faculty, administrators, alumnae and friends to ponder and appreciate the wonderful world of mathematics," said Anderson. "I also hope that all who view the sculpture will begin to grasp the sobering fact that everyone is vulnerable to something terrible happening to them and that we all must learn to live one day at a time, making the very best of what has been given to us. It would be great if everyone who views the Octacube walks away with the feeling that being kind to others is a good way to live."

Click here to view an animation of the sculpture.

Soruce: Penn State

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