Mathematical Theory For Ideal Shape in Nature

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No matter whether they're big, little, long, short, skinny or fat -- classic stalactites have the same singular shape.

Almost everyone knows that stalactites, formations that hang from the roof of caves, are generally long, slender and pointy. But the uniqueness of their form had gone unrecognized.

"There's only one shape that all stalactites tend to be. The difference is one of magnification -- it's either big or it's small, but it's still the same shape," said researcher Martin Short of the University of Arizona in Tucson.

Image: Stalactites in the Big Room of Kartchner Caverns State Park, Benson, Ariz. Photo by Noelle Wilson. Copyright Arizona State Parks

Short and his colleagues have developed a mathematical theory that explains how stalactites get their shape.

"It's an ideal shape in nature and in mathematics that had not been known before," said Raymond Goldstein, a UA physics professor and senior author on the research report. "The Greek philosopher Plato had the concept that there are ideal forms underlying what we see in nature. Although any particular stalactite may have some bumps and ridges that deform it, one might say that within all stalactites is a idealized form trying to get out."

The universality of stalactites had probably been overlooked because the cave formations vary so much in size, said Short, a doctoral candidate in physics at UA.

"The result was a surprise," he said. "We had no idea going into this that we'd find this basic shape."

An article detailing the findings of Short, Goldstein and their colleagues will be published in an upcoming issue of Physical Review Letters.

Other authors on the article are James C. Baygents, a UA associate professor of chemical and environmental engineering; J. Warren Beck, a research scientist in UA's department of physics; David A. Stone, a doctoral candidate in UA's department of soil, water and environmental science; and Rickard S. Toomey, III, science and research manager for Arizona State Parks.

Although people have investigated how cave formations grow, few scientists examined why stalactites have their characteristic shape.

After someone suggested that the tubules David Stone was growing in the laboratory resembled some cave formations, Goldstein became intrigued by caves.

He and his colleagues took a field trip to the famed Kartchner Caverns State Park in Benson, Ariz. and were floored by the variety of forms, especially the ripples many structures possess.

So Goldstein suggested that his student Martin Short investigate the formation of ripples on stalactites.

That task turned out to be extremely difficult, Short said. First he had to learn about the underlying dynamics of stalactite growth.

Stalactites grow when water laden with carbon dioxide and calcium carbonate drips from cracks or holes in the cave's ceiling. As a water droplet hangs from the crack, the carbon dioxide escapes, much as a bottle of sparkling water fizzes when opened. As a result, the calcium carbonate comes out of solution and is left behind as a tiny bit of solid calcium carbonate. As each successive drip flows over the minute mineral deposit, the sequence repeats, ultimately forming a stalactite.

Because the shape stems from the flow of water over the surface of the growing stalactite, the team turned to the field of fluid dynamics. The researchers developed an equation to describe how a stalactite's shape evolves.

"It's a general equation of motion for the growth of stalactites," Goldstein said. "It's a geometric law of motion."

Then the researchers plugged the equation into a computer and asked it to "grow" some shapes. To the team's surprise, no matter what shape was used as a starting point, the computer's formations lengthened and thickened in a universal manner. The results looked strikingly like classic stalactites.

"The computer told us there was something unique to look for, this ideal form," Goldstein said. The researchers then solved their equation of motion and obtained a specific mathematical expression that describes the carrot-like shape of stalactites.

The next step was to test their model against the real thing, so the researchers returned to Kartchner Caverns.

"We spent four hours in the cave with cameras and strobe lights and laptops. We took dozens of pictures," said Goldstein.

Because cave formations are delicate, the researchers could not stomp around measuring the stalactites by hand. Instead, the scientists used lasers to project a pair of green dots onto the stalactites from afar and then took pictures of the stalactites.

The researchers knew how far apart the green dots were, so the dots served as a scale bar for the pictures. Then the researchers could garner the stalactites' dimensions from the pictures.

Back in the lab, the researchers analyzed the actual stalactites and compared their shapes to the ideal form predicted by the mathematics.

The real and the ideal differed by less than 5 percent.

"We calculated the shape mathematically and said, well, we have to go see if this is right," Goldstein said. "And we did. And it was."

Kartchner's Toomey said, "It's cool because the research contributes to learning new things about this cave that apply as well to other caves throughout the world," adding, "Missions of state parks include preservation, understanding and education. To have Kartchner and other state parks available for these types of studies helps further these missions."

Now, Short and Goldstein say, they finally know enough to figure out what gives stalactites their ripples.

Source: University of Arizona