Establishing a new scalar curvature flow method

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Mathematically, is it possible to continuously deform a rough sphere into a perfect sphere? Under what situations can we solve the differential equations?

Professor Xu Xingwang of the Department of Mathematics at National University of Singapore (NUS), along with Dr Chen Xuezhang from Nanjing University of China, has established a new method to tackle this long-standing problem.

What it used to be

Of the various different ways to measure the roughness of the sphere, the scalar curvature measurement has proven to be the weakest one, resulting in a problem commonly known mathematically as the prescribing scalar curvature problem.

The prescribing scalar curvature problem has been an issue for mathematicians for more than half a century. Many great mathematicians have in fact made crucial contributions to it over time, converting the prevalent mathematical problem into the uncovering of a positive solution of elliptic type partial differential equations.

Such equation demonstrates invariance under a non-compact group; a bubbling phenomenon (i.e. a very sharp needle) can occur. Among many mathematicians, it is known that these two difficulties can only surface

in particular manifolds, namely, the prefect sphere case.

In the seventies, French mathematician, T. Aubin, proved an interesting result. He found that if the rough sphere is relatively flat, up to some second lowest tone, it can be deformed to the prefect sphere without changing its angle measure.

In the early nineties, A. Chang and P. Yang, both professors of Princeton University, revisited the mathematical problem and found that if the rough sphere is sufficiently close to the prefect sphere and has no degenerative sharp needle with extra technical assumptions, the deformation is possible. Their argument was made based on the perturbation method and the calculus of variation.

In the recent 10 years, the most significant mathematical achievement is the re-confirmation of the famous Poincare conjecture. Among mathematicians, it is known that the proof is through the powerful Hamilton's Ricci flow, which was introduced nearly 30 years ago. The success in the Poincare conjecture motivated many to re-examine the Ricci flow and apply it to other important mathematical problems.

New mathematical method

On revisiting the age-old problem, Prof Xu and Dr Chen adapted the Hamilton's Ricci flow method to the prescribing scalar curvature problem.

Prof Xu said: "We obtained in a sense the best possible conclusion under the same assumption as the one made by Aubin and Chang-Yang. Basically, we demonstrated that with Aubin's relative flatness assumption plus Chang-Yang's topological assumptions, the problem is solvable. The important observation made in this work is that the relative flatness offered by Aubin is in fact to ensure that the flow can have only

one sharp needle, which in turn makes the blow-up analysis much easier."

Developing a new method attached to the older problem with the same assumptions as what have been done, they came up with a stronger conclusion.

Another tool used in their work was the infinitely dimensional Morse theory. As the theory is well-known, the team found that the challenge was to control the energy level; after all the global existence depends on the initial energy level.

They reported their findings in their paper entitled 'The scalar curvature flow on S^n – perturbation theorem revisited', which was published this February in *Inventiones*, one of the most prestigious journals in the mathematical community.

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